Quadrant II – Transcript and Related Materials

Programme	**: Bachelor of Science (Third Year)**
Subject	**: Electronics**
Paper Code	**: ELD 105**
Paper Title	**: Photonics**
Unit	**: I**
Module Name	**: Concept of Spherical waves**
Module No	**: 02**
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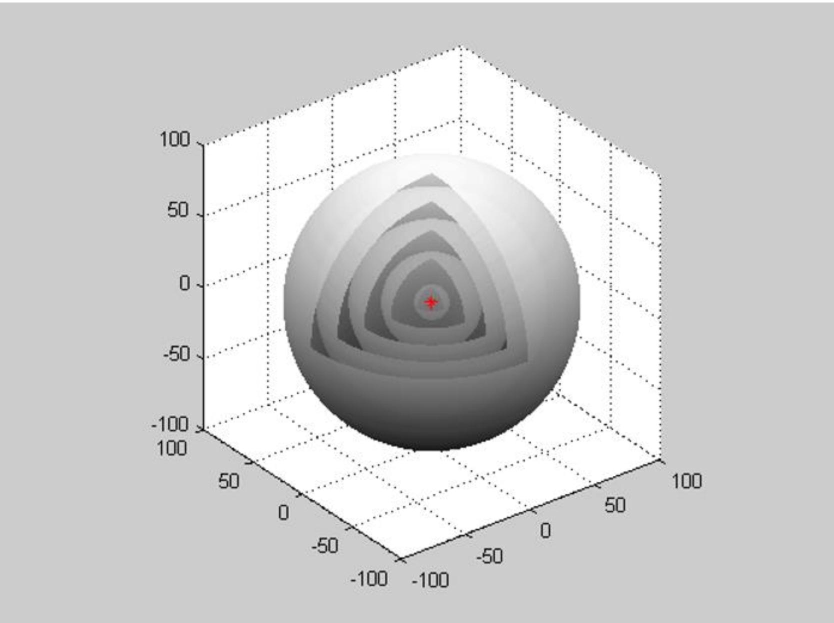
Notes:

What are Spherical waves?

A wave propagating from a source in all direction, it is 3 dimensional.

The medium it travels through is isotropic(constant density), such a wave is called as a Spherical wave.

Eg. Sound/Acoustic wave



Spherical wave equation

- The basic wave equation in free space is given by:

$$\nabla^2 \mathrm{U} - \frac{1}{\mathrm{c}^2}\frac{\partial^2 U}{\partial t^2} = 0 \Rightarrow \nabla^2 \mathrm{U} = \frac{1}{\mathrm{c}^2}\frac{\partial^2 U}{\partial t^2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Where $\nabla^2 \mathrm{U} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z}$

c=Speed of light, U=Field vector,

$$\frac{1}{c^2} = \mu_0 \varepsilon_0 => \quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3x10^8 m/s$$

$\varepsilon_0 =$ Permitivity of free space, μ_0=Permeability of free space

- Where $\varepsilon_0 = 8.85x10^{-12} C^2 N^{-1} m^{-2}$
- $\mu_0 = 4\pi x 10^{-7}\ H/m$
- Let r be the radial distance from the point source to a given point on the waveform.
- Rewriting the wave equation in spherical polar coordinates and considering its spherical symmetric solution we get,

$$\nabla^2 U = \frac{1}{r^2}\ \frac{\partial}{\partial r}\left(r^2 \frac{\partial U}{\partial r}\right) = \frac{1}{r^2}\left(r^2 \frac{\partial^2 U}{\partial r^2} + 2r\frac{\partial U}{\partial r}\right) \text{\_\_\_\_\_\_\_\_(2)}$$

Writing Eq. (2) in compact form:

$$\nabla^2 U = \frac{1}{\mathrm{r}}\left[\frac{\partial^2}{\partial r^2}(rU)\right] \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

Equating right hand sides of Eq. (1) and (3) we have,

$$\frac{1}{\mathrm{r}}\left[\frac{\partial^2}{\partial r^2}(rU)\right] = \frac{1}{\mathrm{c}^2}\frac{\partial^2 U}{\partial t^2} \ or \ \left[\frac{\partial^2}{\partial r^2}(rU)\right] = \frac{1}{\mathrm{c}^2}\frac{\partial^2}{\partial t^2}(rU) \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

rU satisfies the wave equation in variable r. The solution is of the form:

$$rU(r,t) = f(r-ct) \qquad\qquad or$$

$U(r,t) = \frac{1}{r} f(r-ct)$ ___________(5)

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- Eq. (5) represents a general spherical wave traveling outward from the origin at the speed c.
- Because of the factor $\frac{1}{r}$ we see that the spherical wave decays in amplitude as $\frac{1}{r}$, unlike the constant amplitude of plane waves.

Intensity of Spherical waves:

- The Intensity of the wave I *(*power per unit area)

is given by:

$$I = \frac{P}{S}$$

$$Where\ S = 4\pi r^2$$

$$I = \frac{P}{4\pi r^2}$$

- If P is constant and as r increases, the intensity of Spherical wave reduces.
- Suppose we examine 2 different points at radii

r_1 and r_2

Then $I1 = \frac{\mathrm{P}}{4\pi \mathrm{r}_1^2}$, $\quad I2 = \frac{\mathrm{P}}{4\pi \mathrm{r}_2^2}$

If $r2 = 3r1$

$$\frac{I1}{I2} = \left(\frac{r2}{r1}\right)^2$$

$\frac{I1}{I2} = \frac{1}{9}$

$I2$ is reduced by a factor of 9

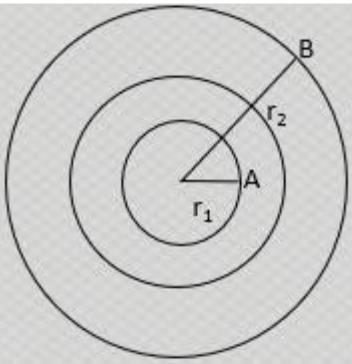

Fig. 3 Spherical waves at 2 different points at radii r_1 and r_2

Amplitude of Spherical waves:

- Let P_1 and P_2 be the average power at points located at distances r_1 and r_2 respectively. Then ,

$P1 = 2\pi^2\rho S1vf2A12$ and $P2 = 2\pi^2\rho S2vf2A22$

Where,

$$\rho - Charge\ Density$$

$$S - Area$$

$$v - velocity$$

$$f - Frequency$$

$$A - Amplitude$$

- Equating P_1 and P_2 we get

$S_1{A_1}^2 = S_2\ {A_2}^2$

$4\pi r_1^2\ {A_1}^2 = 4\pi r_2^2\ {A_2}^2$

$$\frac{A1}{A2} = \left(\frac{r2}{r1}\right)$$

A is inversely proportional to r. Which implies that the amplitude of the spherical waves reduces a factor of r.