Name of the unit is “Interference”

Name of the module is “Thin Film Theory”.

Model number is: 10

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Outline of this module is:

1. Introduction
2. Formation of colours in thin film.

Learning Outcomes module:

At the end of the module, learner will be able to, explain the formation of brilliant colours due to thin films.

Dear students!

Welcome to the session on thin film theory.

When a thin transparent film, like a drop of oil spreading on surface of water is observed by sun light, it exhibits some brilliant colours.

This type of view, usually one observes in rainy season when, the oil spills on roads.

Similarly, one observes different colours in soap bubble.

These colours are mainly due to the phenomenon of ‘interference in thin film’.

In first case, a thin oil film is formed on the surface of water and in second case, the thin film of soap solution is formed. And the interference in thin film due to division of amplitude takes place.

You have already studied the different techniques of obtaining interference, in last module.

In division of amplitude method, the incident wave continuously gets divided between reflected & refracted rays due to its two bounding surfaces. The reflected rays emerging out from the top surface and interfere to produce the interference pattern. Similarly, transmitted rays emerging out from the bottom surface also interfere to produce the interference pattern.

One can observe the interference so formed, from the top of the film OR from the bottom of the film. The former is called 'Interference in reflected system' and later 'Interference in Transmitted system'.

<u>Formation of colours in thin film</u>

- Consider a thin transparent film of uniform thickness (t) bounded by two parallel surfaces XY & WZ. Let μ be the refractive index of the medium of the film.
- When a light is incident on the transparent film, a small portion gets reflected from the top surface and the major portion is transmitted into the film. Again, a small portion of transmitted component is reflected back into the film by the bottom surface and the rest of it, is transmitted from the lower surface of the film.
- Thin film transmits, incident light strongly and reflects weakly. Hence after two reflections, the intensities of reflected light become negligibly small. Therefore, we consider only first two reflected rays.

Let monochromatic source of light, of wavelength λ be incident along AB. This ray will get partially reflected along BC (this ray is represented as ray 1) and partially refracted along BF. Partially refracted ray further suffers partial reflection along FD and partial refraction along FK. Now the partially reflected ray undergoes partial reflection and refraction through the film and emerges along DE. Let us represent this ray as, ray 2.

It is very clear that ray 1 & ray 2 are derived from a single incident wave. Therefore, they are coherent waves and can produce interference, when they overlap.

Let us see the condition, when the two rays' overlaps.

(i) Geometrical path difference.

Draw HD perpendicular to BC.

Consider the formation of ray 1 & ray 2 from point source B. The ray 1 starts from B and reaches to H; whereas ray 2, travel along BF and along FD within the film. And, after point D, both the rays travel equal path. This means ray 1 travels distance BH, while ray 2 travels distance (BF+FD) before travelling equal distances.

Hence geometrical path difference between the two rays is

$$L = (BF + FD) - BH$$

(ii) Optical path difference (OPD)

By definition, OPD = μ. L. Therefore, OPD due to two rays will be:

$$\Delta_a = \mu.L = \mu(BF+FD) - 1.BH \quad \ldots\ldots(1)$$

We draw the normal, (shown in green colour) in the figure.

w.r.t. normal, let, i be the angle of incidence

Using the law of reflection, angle of reflection = angle of incidence.

Consider two triangles BFG & DFG. They are similar.

$$\therefore \; BF = FD$$

Now using the trigonometric ratio, we get,

$$BF = \frac{t}{\cos r}$$

In triangle BDH, we can show that $BH = 2\, t.\tan r \, \sin i$

Using the Snell's law, we can write $\boldsymbol{\sin i = \mu \,.\, \sin\ r}$

On substituting we get:

$$BH = \frac{2\mu t \sin^2 r}{\cos r}$$

Finally substituting above in equation 1 we get,

$$\Delta a = 2\mu t \cos r$$

This is the optical path difference, we observe, due to traveling of the rays through the air and through the film.

Apart from this observed optical path length (we will refer this, as apparent path length), we also have the phase change due to reflection.

Whenever a reflection of light is accompanied by a refraction from a rarer to denser medium, the reflected ray suffers a phase difference of π, that is inversion takes place and therefore there is a path difference of $\lambda/2$ between the two rays; whereas there is no path difference due to reflection at the boundary from denser to the rarer medium. Also there is no path difference due to reflection at any stage.

The phase change takes place only between ray1 and the incident ray. No phase change takes place due to, ray 2. Therefore, true optical path difference between the ray 1 and ray 2 will:

$$\boldsymbol{\Delta_t = 2\,\mu\, t\ \ \cos r\ \ -\ \lambda/2}$$

This is the optical path difference between two interfering rays.

Now, when these two rays, come out of the film, there can be different conditions of path difference.

Let us see, what are the conditions to generate a maximum or a minimum. That is, a brightness or darkness.

Conditions for Maxima (Brightness) And Minima (Darkness).

MAXIMA:

case 1 : Suppose two rays overlaps such that 1st crest of ray 2 falls on the 1st crest of ray 1. Then OPD = zero (i.e. no path difference). The two waves interfere constructively and produce maximum illumination (Brightness).

case 2 : The two rays overlaps such that 1st crest of ray 2 falls on the 2nd crest of ray 1. Then OPD= one wave (λ). The two waves interfere constructively and produce maximum illumination (Brightness).

case 3 : The two rays overlaps such that 1st crest of ray 2 falls on the 3rd crest of ray 1. Then OPD = two wave (2λ). The two waves interfere constructively and produce maxima.

We generalise above two cases and say that: If the OPD between the two rays equals integral number of full waves, they meet in phase and produce MAXIMA.

Mathematically, we write this condition as:

$$\Delta_t = 2\mu\ t \cos r - \lambda/2 = m\lambda$$

$$\therefore 2\mu\ t \cos r = m\lambda + \frac{\lambda}{2}$$

i.e. $2\mu\ t \cos r = (2m+1)\frac{\lambda}{2}$ … (I) *where m= 0,1,2,….*

condition for brightness or maxima.

MINIMA (DARKNESS):

<u>case 1</u> : Suppose two rays overlaps such that 1st crest of ray 2 falls on the 1st trough of ray 1,then two waves cancel each other leading to destructive interference. This causes darkness.

From the figure, we can say, the OPD for this condition is equals the *half wave* i.e. ($\lambda/2$).

<u>case 2</u> : The two rays overlaps such that, 1st crest of ray 2 falls on the 2nd trough of ray 1. The two waves cancel each and produce Minima. From the figure, the OPD = *three half waves* ($3\lambda/2$).

Now we generalise, and say that: If the OPD between the two rays equals odd integral number of half wave, they meet each other *in opposite phase* and cancel and produce no illumination (i.e. produces darkness).

Mathematically, we write this condition as:

$$\Delta_t = \quad 2\,\mu\ t \cos r - \lambda/2 = (2m+1)\lambda/2$$

$\therefore 2\,\mu\ t \cos r = 2m\,\lambda/2 + \lambda/2 + \lambda/2$ i.e. $2\,\mu\ t \cos r = m\lambda + \lambda$

$\therefore \quad 2\,\mu\ t \cos r = (m+1)\lambda$

The phase relationship of interfering waves does not change if <u>one</u> full wave is added or subtracted. Therefor we replace *(m+1)* with *m*

Hence above equation, can be written as

$$2\,\mu\ t \cos r = m\lambda \qquad \text{(II)}$$

where $m = 0,1,2,\ldots$.condition for darkness

This is the condition for the two rays to form the MINIMA.

Let us summarise both the conditions.

$$2\mu\ t\cos r = (2m+1)\frac{\lambda}{2} \qquad \ldots \text{ (I) Brightness}$$

$$2\mu\ t\cos r = m\lambda \qquad \ldots \text{ (II) Darkness}$$

Physical interpretation:

(1) When a parallel sided film, of uniform thickness (t), is illuminated by monochromatic source of light (λ), the whole film will appear either bright OR dark, depending upon whether condition (I) OR (II) above is satisfied. (since *μ , t, r, λ* all remains *constant)*

(2) When a parallel film of uniform thickness(t), is illuminated by white light, then λ will vary.

Now, suppose the condition (I) is satisfied for λ_R, the whole film will appear red in colour. (since *μ , t, r* all remains constant)

$$2\mu\ t\cos r = (2m+1)\lambda_R/2$$

(3) Now, if parallel beam of white light falls on a parallel film, such that, condition (II) is satisfied for blue colour (λ_B) then, all other colours except blue will be reflected from the film.

$$2\mu\ t\cos r = m\lambda_B$$

Therefore, the film will appear uniformly coloured with blue being absent.

(4) Similarly, we can explain any other colour from the film.

(5) Now let us come to the major portion of our topic. That is a film of non-uniform thickness, like a drop of oil spreading on the surface of water. Here thickness of the film will be different for at different points, because oil drop spreading on the surface for water will have a random thickness.

(i) Suppose we have a point where the thickness of the oil film is t1 and λ_R satisfies condition (I).

Therefore, light reflected from the point of thickness t1 will be red in colour and all along the contour of the film, where the thickness is t1, red light will be reflected from the oil film. You can see the top of the film, the red colour being reflected due to drop of oil.

(ii) Likewise, we can consider second thickness t2 which satisfies condition (I) for green colour ($\lambda_{G,}$).

The light reflected from a film due to thickness t2 will be green in colour and all along the film, where the thickness is t2, will reflect green in colour or you will get green ring, the one which can be seen on the display.

(iii) Likewise, we can explain all other colours.

This explain why we get different colours, when a drop of oil spreads on the surface of water.

Conclusion

Colours due to thin films are observed depending upon whether the condition for intensity maxima or intensity minima are satisfied.

This is a beautiful image of the colours observed in rainy season, due to drop of oil spreading on the surface of water.

These are the books, I have referred.

Thank you very much.