Welcome back students and viewers. I'm Hussain Yasser Razak from M. E. S. College of Arts and Commerce and we are doing Mathematical economics I: ECD 117. This is unit 1 basic concepts in mathematics. This module is systems of simultaneous equations. This is a continuation of the previous module and this is module #7. In the outline we have an introduction to simultaneous equations and we have solving simultaneous equations with two variables. The learning outcomes are students will be able to understand the concept of simultaneous equations and solve simultaneous equations for two unknowns. In the introduction, first, let's understand what conditions need to be fulfilled in order for an equation to be considered a simultaneous equation. A simultaneous equation system exists if. More than one functional relationship exists between a set of specified variables. And all relationships are linear in nature. These points are very important. The first point is that more than one functional relationship exists between a set of specified variables. The same specified variables and all relationships are in a linear nature. Solving simultaneous equations, we can have different methods to do this. The first method that we will be looking at is the graphical method followed by equating two equations followed by substitution method. And the last method would be eliminating by equating coefficients but will get into depth with each of these methods and let's see how we get to solving around the simultaneous equations. So let's look at the first one, the graphical method. This is something that we do in Economics, very frequently. . We don't actually get into depth into mathematics into solving these kinds of problems, but essentially mathematics is an important part of solving these problems graphically. So let's say we have a demand equation, QD is equal to 60 - 10P like we saw in the previous module, and we have a demand schedule here, which hasn't been filled up yet. So we have prices 5, 4,3,2,1. And we are substituting these values of prices in this particular quantity demanded equation and we fill in the particular table just like that. OK, if you have missed the part, how we calculate these particular values, please go back to the previous module and get into learning how we have done that. So we've got a demand schedule ready. Now we have price 5, 4,3,2,1 and we have the quantity demanded at 10, 20, 30, 40, 50. Is there an inverse relationship? Yes! indeed there is an inverse relationship. But to solve this, simultaneous equations will require another equation, right? There needs to be at least 2 functions, so in this case let's take the counterpart. That's a supply equation, and in this case you have Qs= 10 P. If you note this particular equation does not have an intercept term, but nevertheless has two variables Q and P and. If you note the demand and the supply curve, both of them have Q&P as their variables. Hence it establishes. A system of simultaneous equations. So let's look at the same thing that we did in the demand equation. So we have Qs is equal to 10 *1. That's the first value of P and we get 10 and doing the same thing for all the other values of P will have our supply schedule . So we have the demand schedule. We have a supply schedule . What we gotta do now is just like in the previous module , we're going to plot two graphs, one for the demand and one for the supply. So let's head onto it. This is your demand curve. It is a downward sloping curve like all of you must be knowing already and the supply curve would be an upward sloping curve just like that. But how do we find an equilibrium in this particular method? We have two separate graphs on two different sheets and two different curves. How do we find equilibrium? What we do is we superimpose one graph on the other. In other words, you draw third graph if you're doing it on paper. And you superimpose one on the other. That means now you have both your graphs, the supply and the demand curve on the same graph, and you can see how they intersect at a particular point and that point, My friends is (30,3), That's Quantity 30 and Price 3. In a moment we'll be looking at how we get those particular points. You can a get look at this particular point that's come up on your screen now if you haven't caught a glimpse of it, again, that's three there. That's 30 there, right? So we relate them. We have the dotted lines coming up to the equilibrium point. That's your equilibrium, (30,3) and Once again, look at that (30,3). That's the equilibrium point. So the quantity demanded at the equilibrium level will be 30, and the price at the equilibrium level will be 3. And there we have it. The equilibrium . This method was your graphical method of finding the Q and P at equilibrium. Let's move on to the next method, and this method is called equating two equations.So just as the name suggests you know that there have to be 2 equations and we are going to equate them. So we have the first equation that is very same one that we used in the previous method . That's Q is equal to 60 - 10 P. I've gotten rid of the Qd now because by now we understand that this is a demand equation . Q is equal to 60 - 10 P. And I've slightly modified the supply equation. It is not P is equal to Q by 10 and if notice now the demand equation is in terms of Q and the independent variable P on the right hand side where as the supply side you have the price on the left hand side and you have quantity on the right hand side. What you need to do first is step one is express each equation in terms of the same variable. So what I do is I have my Q is equal to 60 - 10 be there. I'll adjust the supply equation and I'll get Q is equal to 10P. Straightforward math we've multiplied that to P and it's become Q is equal to 10 P. Now we have both equations established. On the same grounds we have Q On the left hand side, we'll be having P on the right hand side. Now you know that if Q is equal to A and Q is equal to B, that means A and B must be equal, shouldn't it? Therefore we take the second step that substitute one equation with the other and then we solve them. But if you look at this particular place now, I have replaced my demand equation which had Q here with my entire answer for Q, that's 10P here right there, OK? And with this what has happened… The equation that had two variables now has only one variable and all of us know how to solve this right? So we have the next step that's 20P. What's happened ? Is this particular negative 10 P is going to the other side the actual way that's done is what you've done is you have added 10 P to both sides. And because , you added 10 P to both sides, it got cancelled from here and that's got added on there . That's the actual process of the way it's done, so you have 20 P is equal to 60 and then finally we know that B is equal to 3. Once you've got P is equal to three, it's pretty straight forward. From there. All you need to do is take that P is equal to three. Step three, substitute the obtained value in any original equation could be the demand or could be the supply one. I'm going to do both for you. So we take the demand equation . We plug in the value right there for P. And i have circled it for you . And that turns out to be Q is equal to 60 - 30, which turns out to be Q is equal to 30. Let's try to the supplier side, maybe I'm wrong, maybe we get a supply equation that doesn't show a Q equal 30. Let's check it out so we have a supply equation, equal to P is equal to Q up on 10 and substitute P is equal to three. So we get that we have 3 into 10 Q, Once again we have 30. So the answer is absolutely right , whether you substitute from the demand equation or the supply equation, you will get Q is equal to 30. Finally, we have the third method. That's a substitution method. In the substitution method we're going to do is substitute . We're going to take two different demand and supply equations. OK, not the ones that we took in the previous case, and in this case we have the demand equation. Q is equal to 80 - 4 P and supply equation which is Q is equal to negative 20 + 8 P and what we're going to do here is we're going to transform the equation which we're already mentioned in the same term into the different term . For example, this right here will be expressed in terms of Q. Now we're going to convert it and express it in terms of P&Q. Now the reason we're doing that is because we want to substitute this entire term. Into the demand equation andI'll show you how it will look. Step one is to transform . So we've transformed here once we've transformed. Step 2 is substitute , so I've taken this particular value that I've got for P and I've replaced it right here in the demand equation in place of P. Be careful of the brackets. Look there are double brackets there OK, and see that you've done that properly, or else you will cancel off the wrong numbers and get a wrong answer. OK, be careful of the brackets. Once you've done that, step three is solve. Let's just like the previous problem that we discussed. Now it has the same terms Q&Q there, right? So it's pretty straightforward and we all know now from the previous slide how we got .5 Q to the left hand side, right? So that ends up becoming 1.5. Q is equal to 70. Solve that out. You get Q is equal to 46.66 What do we do now? We take the Q is equal to 46.66 and we re-substituted into the next particular equation and we solve that equation we have P is equal to 8.33. Let's check this out in a graphical form and there we have our equilibrium point . That's the demand equation and supply equation and we find that Q is equal to indeed 46.66 and P is equal to 8.33. That's the equilibrium point and we come to the last and final method of elimination by equating coefficients. We take the same 2 demand equations that we had in the previous case. Step one is transform. In this transformation, we're going to do is take the variables to the left hand side and keep the constant numbers on the right hand side, just like I've done here in the next step will. Number them as equation 1 and equation 2. What we want to do is we want to equate these two coefficients. Of course in this example we can see it would be very simple to equate these two since they're one. But let's do something difficult. So you understand the process. So the way is to identify the variable, the coefficient that you want to eliminate . In this case, I've identified four and eight. I want to equate that and since I want to equate that, I have to multiply this whole equation by 8 'cause it's going to make it 32. And I need to Equate the equation 2 with four with the, multiply that with four here. And once you've done that, you see the resulting equation gives you something like this equation 3 and equation 4. And finally we can eliminate it. In this case we have opposite signs already, so it's pretty much simple to cancel off in case you don't have opposite signs, you need to change the sign of any one of the equations throughout the equation, not just in one place, so this cancels off. That comes on the Q is equal to 46 , just like the previous method. And once again we have the equilibrium that we saw using the last method, right? So we have the same answer. So guys thank you for joining me. These were the few methods to solve simultaneous equations. These are some reference books that you can use . Thank you very much. See you again in the next module.