Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (First Year)

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Paper Title: DSC 1B Matrices and Linear Algebra

Unit: Unit 4 Vector Spaces

Module Name: Vector Subspaces-Part 5

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Notes:

Finite Linear Combination

Definition:

Let V be a vector space over a field F.

Letv_1, v_2 , v_3,…..,$v_n$be elements (vectors) of V.

A vector $v \in V$ is called a finite linear combination of vectors v_1, v_2 , v_3,….., v_n if there exists scalars a_1, a_2,……, a_n in F such that $v = a_1 . v_1 + a_2 . v_2 + a_3 . v_3 + \ldots\ldots + a_n . v_n$

For example 1 (Linear Combination in R^3)

Suppose we want to express v= (3,7,-4) in R^3 as a linear combination of vectors v_1=(1,2,3), v_2= (2,3,7), v_3= (3,5,6)

Then we find scalarsa_1, a_2, a_3 such that

$$v = a_1 . v_1 + a_2 . v_2 + a_3 . v_3$$

i.e. (3,7,-4) = a_1.(1,2,3) + a_2 .(2,3,7) +a_3.(3,5,6)

i.e. (3,7,-4) = (a_1+2a_2 + 3a_3, 2a_1+ 3a_2+5a_3, 3a_1+7a_2+ 6a_3)

This gives

$$a_1 + 2a_2 + 3a_3 = 3$$
$$2a_1 + 3a_2 + 5a_3 = 7$$
$$3a_1 + 7a_2 + 6a_3 = -4$$

Reducing the system to **row echelon form** gives

$$a_1 + 2a_2 + 3a_3 = 3$$
$$-a_2 - a_3 = 1$$
$$a_2 - 3a_3 = -13$$

Then gives

$$a_1 + 2a_2 + 3a_3 = 3$$
$$a_2 + a_3 = -1$$
$$a_3 = 3$$

Back substitution gives a_1=2, a_2= -4, a_3=3

∴ We can write

(3,7,-4) = 2.(1,2,3) -4 .(2,3,7) +3.(3,5,6)

Thus a vector $v = (3,7,-4)$ in R^3 is expressed as a finite linear combination of vectors $v_1 =$ (1,2,3),v_2=(2,3,7), v_3=(3,5,6)

Example 2 (Linear combination of polynomials)

Suppose we want to express the polynomial

$v = 3t^2 + 5t - 5$ as a linear combination of the polynmials $p_1 = t^2 + 2t + 1$, $p_2 = 2t^2 + 5t + 4$,

$p_3 = t^2 + 3t + 6$

We find scalars a_1, a_2, a_3 such that

$v = a_1p_1 + a_2p_2 + a_3p_3$

i.e. $3t^2 + 5t - 5 = a_1(t^2 + 2t + 1) + a_2(2t^2 + 5t + 4) +$

$a_3(t^2 + 3t + 6)$

Expanding the right –hand side, we obtain

$3t^2 + 5t - 5 = (a_1 + 2a_2 + a_3)\,t^2 + (2a_1 + 5a_2 + 3a_3)t + (a_1 + 4a_2 + 6a_3)$

Set coefficients of the same powers of 't' equal to each other

Thus we get

$a_1 + 2a_2 + a_3 = 3$

$2a_1 + 5a_2 + 3a_3 = 5$

$a_1 + 4a_2 + 6a_3 = -5$

i.e. We get the following system of 3 linear equations in three unknowns:

$a_1 + 2a_2 + a_3 = 3$

$2a_1 + 5a_2 + 3a_3 = 5$

$a_1 + 4a_2 + 6a_3 = -5$

Reduce the above system of linear equations to row echelon form, we get

$a_1 + 2a_2 + a_3 = 3$

$a_2 + a_3 = -1$

$2a_2 + 5a_3 = -8$

Then gives

$a_1 + 2a_2 + a_3 = 3$

$a_2 + a_3 = -1$

$a_3 = -2$

The above system is in **triangular form** and has a solution

Back substitution gives the solution

$a_1 = 3$, $a_2 = 1$, $a_3 = -2$

Thus we get,

$3t^2 + 5t - 5 = 3(t^2 + 2t + 1) + 1(2t^2 + 5t + 4) +$

$-2(t^2 + 3t + 6)$

Thus a polynomial $(3t^2 + 5t - 5)$ is expressed as a finite linear combination of polynomials $(t^2 + 2t + 1)$,
$(2t^2 + 5t + 4)$ and $(t^2 + 3t + 6)$

Example 3:(Linear Combination of Matrices**)**

Write the matrix $E = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$ as a linear combination of the matrices $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$,
$C = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$

Solution: We find scalars a_1, a_2, a_3 such that

$$E = a_1.A + a_2.B + a_3.C$$

$$= a_1 \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + a_3 \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} a_1 & a_1 + 2a_3 \\ a_1 + a_2 & a_2 - a_3 \end{bmatrix}$$

Equating the matrices and solving the system of equations,

$a_1 = 3$

$a_1 + a_2 = 1$

$a_1 + 2a_3 = 1$

$a_2 - a_3 = -1$

we get the solution as:

$a_1 = 3$, $a_2 = -2$, $a_3 = -1$

Hence $E = 3A - 2B - C$

Thus matrix E is expressed as a linear combination of matrices A, B, C