Quadrant II – Transcript and Related Materials

Programme	**: Bachelor of Science (First Year)**
Subject	**: Physics**
Paper Code	**: PYC 102**
Paper Title	**: Heat & Thermodynamics and Properties of Matter & Acoustics**
Unit 4	**: Second Law of Thermodynamics**
Module Name	**: Reversibility of Carnot's Cycle & Carnot Theorem, Coefficient of Performance of Refrigerator**
Module No	**: 21**
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Notes

Reversibility of Carnot's cycle:

The four stages through which the working substance is taken are perfectly reversible because it is assumed in describing the cycle that;

(i) The processes are carried out extremely slowly.

(ii) The source and the sink have infinite thermal capacity so that their temperature do not change due to exchange of heat with the working substance.

(iii) There is no friction between the piston and the wall of the cylinder.

(iv) The piston and the side of the cylinder are impermeable to heat.

(v) The base of the cylinder is a thin sheet of a good conducting material.

However, these are all ideal conditions which cannot be completely realized in a practical heat engine.

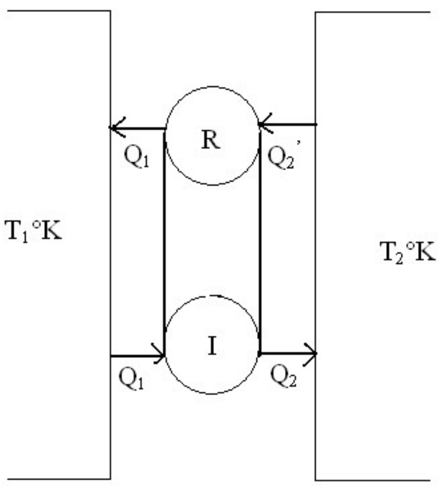
Carnot's Theorem:

Statement: No engine can be more efficient than a reversible engine working between the same two temperatures.

Proof:

Consider a reversible engine R and an irreversible engine I coupled in such a way that I works in the forward direction and drives R in the reverse direction. i.e. R works as an refrigerator as shown in fig. below.

If possible, let I be more efficient than R.



Let I abstract heat Q_1 from the hot reservoir at T_1°K, convert part of heat into work W and reject heat Q_2 to the cold reservoir at T_2°K.

Thus, $W = Q_1 - Q_2$ -------------------------------(1)

Or, $Q_2 = Q_1 - W$ -------------------------------(2)

Similarly, let R abstract heat Q_2' from the cold reservoir T_2°K and W' be the work done on it – as it is working as a refrigerator. For the sake of simplicity, we shall assume that the heat rejected by R to the hot reservoir at T_1°K is equal to Q_1, the same as that taken from it by I.

Therefore,

$W' = Q_1 - Q_2'$--------------------------------------(3)

$\therefore Q_2' = Q_1 - W'$ ------------------------------------(4)

As we have assumed that I is more efficient than R;

$$\frac{W}{Q_1} > \frac{W_1'}{Q_1}$$

$Q_1 - Q_2 > Q_1 - Q_2'$

Or $Q_2' > Q_2$ ---(5)

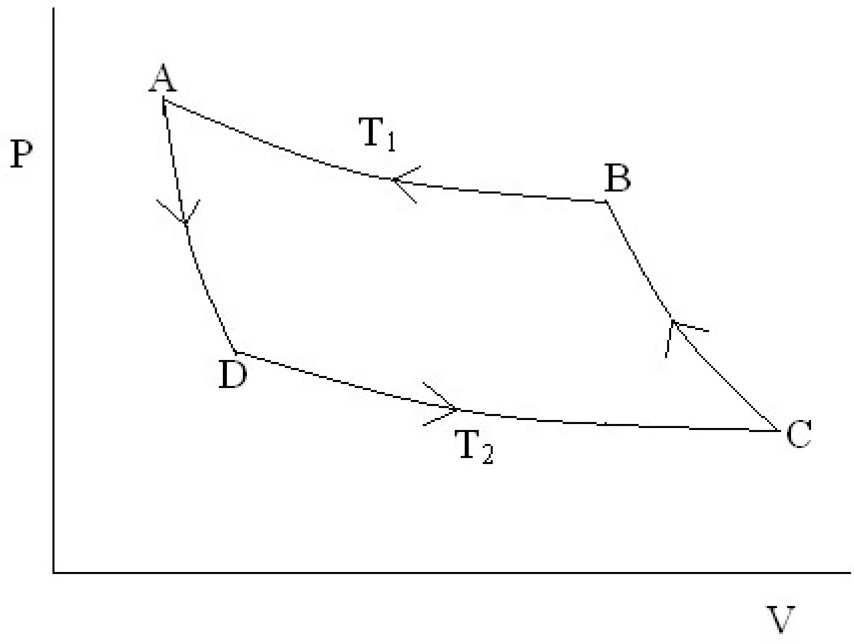
This shows that heat lost by the cold reservoir Q_2' to R is more than the heat Q_2 gained by it from I. Thus, the cold reservoir continuously loses an amount of ($Q_2' - Q_2$) and the work is being done, without producing any change in the hot reservoir. Thus, heat is continuously extracted from a single body and converted into work which is contrary to experience and violates second law of thermodynamics. Hence our assumption that I is more efficient than R is wrong. Therefore, I cannot be more efficient than R.

[Comparison between engine and refrigerator]

The working of an ideal heat engine and that of an ideal refrigerator are just reverse of each other. In a heat engine, the working substance extracts heat from a hot reservoir, some of which is utilized in doing external work and rest of it is rejected into cold reservoir (atmosphere). In a refrigerator, an external agency (an electric motor) does mechanical work on the working substance which extracts heat from a cold reservoir (cold storage space inside the refrigerator) and the whole amount of energy is rejected to a hot reservoir (outer atmosphere). This is the working principle of the ideal refrigerator.

Stages in refrigeration plant and P-V diagram:

To understand the thermodynamical aspect of the refrigeration plant, we follow the Carnot cycle in reverse direction as shown in fig. below.



The closed figure ABCD represents refrigeration cycle which consists of four parts AD, DC, CB, BA. Each part corresponds to a Stage.

Stage 1: During adiabatic expansion AD, work is done by the gas. Its temperature falls from T_1 to T_2.

Stage 2: During isothermal expansion DC, an amount of heat Q_2 is absorbed from the cold reservoir of lower temperature T_2 and the work is done by the gas.

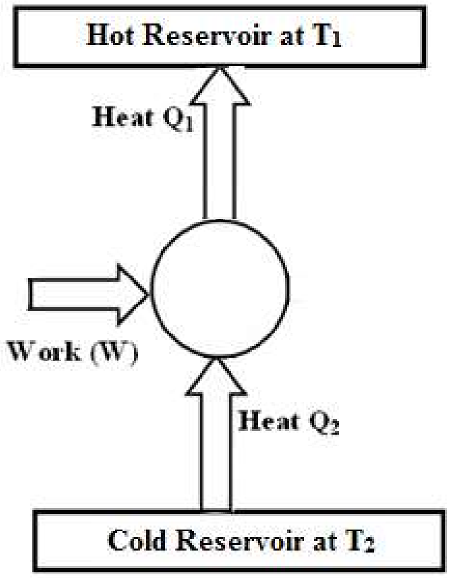
There is no change in temperature.

Stage 3: During adiabatic compression CB, work is done on the gas with an external agency (electric motor), so that its temperature rises from T_2 to T_1. No heat is either absorbed or rejected.

Stage 4: During adiabatic compression BA, an amount of heat Q_1 is transferred by the gas to the outer system at a higher temperature T_1. Some work I done on the gas. The original state is restored. One cycle of refrigeration is complete.

Principle of Refrigeration:

The schematic representation of the refrigerator is shown below:



Heat Q_2 is absorbed from a cold reservoir at temperature T_2. Work W is done with an external agency (electric motor) on the working substance. Heat ($Q_1 = Q_2 + W$) is transferred by the working substance to the hot reservoir at temperature T_1. Thus work is always necessary to transfer heat from a cold reservoir to a hot reservoir. A refrigerator cannot work without the supply of electrical energy.

Coefficient of Performance (C.O.P.):

Let W be the total amount of work done on the gas during the refrigeration cycle. There is no change in the internal energy as the original state is restored. The net amount of energy absorbed is ($Q_2 - Q_1$).

Applying the first law of thermodynamics, $Q_2 - Q_1 = 0 - W$

$\therefore W = Q_1 - Q_2$

Thus the heat Q_2 is transferred from the cold reservoir at temperature T_2 to the hot reservoir at temperature T_1. Such a device is called refrigeration and the working substance is known as refrigerant.

A good refrigerator is one which removes more heat Q_2 from the cold reservoir with less expenditure of work W. The capacity to do this task is measured in terms of C.O.P and is defined as; the ratio of heat removed (Q_2) from the cold reservoir at temperature T_2 to the corresponding work done (W) on the working substance.

i.e. $C = \frac{Q_2}{W}$

For Carnot refrigerator, we have,

$$C = \frac{Q_2}{Q_1 - Q_2} \quad since\ W = Q_1 - Q_2$$

$$Since, \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$C = \frac{T_2}{T_1 - T_2} = \frac{1}{\frac{T_1}{T_2} - 1}$$

The efficiency of Carnot engine is given by;

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

$$Since 1 - \eta = {T_2}/{T_1}$$

$$\therefore C = \frac{1 - \eta}{\eta}$$