Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (Third Year)

Subject : Physics

Course Code: PYC 108

Course Title : DSC: Atomic and Molecular Physics

Unit : X-ray spectra

Module Name: Characteristic spectrum, Mosley's law

Name of the Presenter: Mahendra R. Pednekar

Notes :

X – Ray Spectra

X – rays are electromagnetic radiations of very short wavelength (wavelength range = 0.01 A^0 to 100 A^0).

When X – rays from an X - ray tube are analysed using a crystal spectrometer, it is observed that the X – ray beam from a target consists of two distinct types of spectra.

1. A continuous spectrum
2. A sharp line spectrum superimposed on the continuous spectrum.

The figure shows a typical X – ray spectrum, Where the intensity of X – rays is plotted against the wavelength, the curve for tungsten (W) shows the continuous spectrum. Here the X – ray tube is operated at 35 kV with the tungsten as the target. The other curve is for molybdenum (Mo). This curve shows two sharp lines characteristic of element molybdenum, which are superimposed on the continuous spectrum. These lines are known as K_α and K_β lines of molybdenum. To get similar lines for tungsten, it is observed that the tube has to be operated at a very high voltage of about 70 kV.

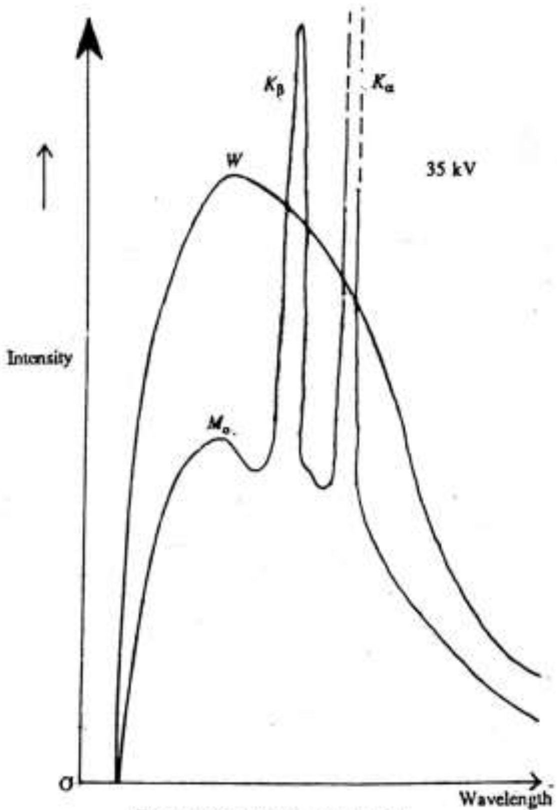

Fig. 5.1 Typical X – ray spectra

Characteristics of the spectrum:

1. The spectrum is in general continuous. It has a minimum wavelength limit λ_{min} called the Duane – Hunt limit. This short wavelength limit is characteristic of the operating voltage and is independent of the material of the target as is evident from the fact that both tungsten and molybdenum have the same short wavelength limit.

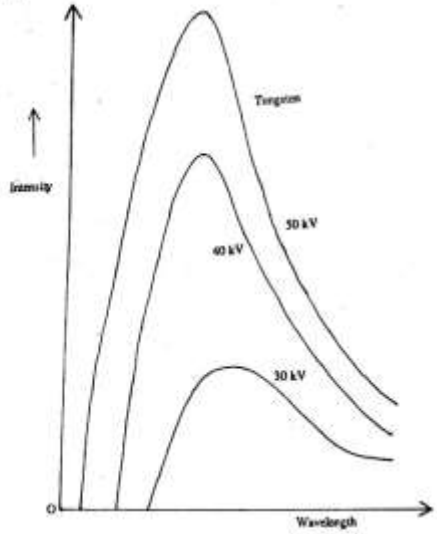

Fig. 5.2 Variation of the Duane-Hunt short wavelength limit with voltage

2. Keeping the target material same , if the operating voltage is increased, then it is observed that λ_{min} i.e. the short wavelength or minimum wavelength limit, decreases. It is found that λ_{min} is inversely proportional to the applied voltage.
3. If the voltage at which the tube is operated is increased above a certain minimum voltage, called the critical voltage, sharp peaks appear depending upon the nature of the target material. These peaks correspond to definite wavelengths which are characteristic of the material of the target and hence the corresponding lines are called the characteristic lines.

Though the continuous spectrum for all targets is similar in nature, its intensity depends upon the atomic number of the target. The intensity of a given spectrum is the area under the curve and it is proportional to the atomic number of the target. Also for a given material(Z -constant), the intensity is proportional to the square of the voltage applied across the tube and the maximum of the intensity curve shifts towards shorter wavelength as the voltage applied across the tube is increased.

on the other hand the characteristic spectra appear only when the voltage across the tube has a certain minimum value which is necessary to excite the characteristic lines depends upon the atomic number of the target and increases with increase in Z.

Continuous spectra

when charged particle is accelerated, it emits electromagnetic radiation. The intensity of the radiation depends upon the magnitude of acceleration. The production of a continuous spectrum is a result of this radiation. In an x-ray tube, the electrons are accelerated due to the high potential difference applied across electrodes. These high velocity electrons are incident on the target material and due to their large velocity penetrate a few layers of the target material. The nuclei of the atom exert electromagnetic forces on the incident electrons. The velocity of the electrons is therefore reduced to a great extent resulting in large retardation. Due to this retardation the electrons emit electromagnetic radiation. This is known as Bremsstrahlung radiation. The radiation gives rise to a continuous spectrum since not all electrons are retarded uniformly and the radiation of the electrons is different in different parts of the tube.

The existence of the short wavelength limit (Duane-Hunt limit) of an X-ray spectrum, which is the same for all elements for a given operating voltage, can be explained using the quantum theory. The quantum theory treats X-rays as consisting of photons, the energy of these photons is hν, where ν is the frequency of radiation. The kinetic energy of the electrons accelerated from rest through a potential difference V (in volts) is eV , which is the energy with which they are incident on the target. Hence in general, the energy of the photon (hν) must be less than the kinetic energy of the incident electron, i.e.

$h\nu < eV$

The maximum energy a photon can have is given by

$h\nu_{max} = eV$

$h \frac{c}{\lambda_{\min}} = eV$ $\qquad\qquad$ $\nu_{max} = \frac{c}{\lambda_{\min}}$

therefore $\lambda_{min} = \frac{hc}{eV}$

which implies $\lambda_{min} \propto \frac{1}{V}$

$h = 6.63 \times 10^{-34}$ Js,

$c = 3 \times 10^{8}$ m/s

$e = 1.6 \times 10^{-19}$ coulomb

1 eV = 1.6×10^{-19} joule

$\lambda_{min} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1{,}6 \times 10^{-19}}$ in metre

$\lambda_{min} = \frac{12400}{V}$ in A^0

The Mosley Law

When a systematic study of X-ray radiation from various elements was made, Mosley found that the wavelengths of characteristic X-ray lines change smoothly as one goes from element to element. The X-ray spectrum consisted of discrete lines superimposed on the continuous X-ray spectrum. The wavelength of the X-rays decreases as Z increased, indicating that the frequency of the X-rays increased with increase in Z. when a plot of $\sqrt{\nu_{K_\alpha}}$ (the square root of frequency of K_α radiation) versus atomic number Z was made, it was expressed as Mosley law as

$\sqrt{\nu_{K_\alpha}}$ α Z

Where ν_{K_α} is the frequency of K_α lines for the various elements .

$\nu_{K_\alpha} = \frac{3}{4}$ c R (Z – 1)^2 where the constants c and R are the speed of light in vacuum and Rydberg constant respectively.

Mosley drew the following important conclusions from the study of X ray spectra of different elements.

1) Elements in the periodic table should be arranged according to their increasing atomic number Z, and not their atomic weight A.
2) The results also showed how the elements with smaller atomic weight should be placed after the elements with greater atomic weights if the elements with greater atomic weights have smaller Z. For example, cobalt (Co) (Z=27) which has atomic weight 58.9 comes before nickel (Ni) (Z=28) which has atomic weight 58.7.
3) Many elements which were missing in the periodic table could also be predicted. Thus Mosley predicted the occurrence of scandium (Sc) (Z-21) and promethium (Pm) (Z=61), which were discovered later.

With the discovery of high resolution spectrometers, the fine structure of K series lines, like doublets $K_{\alpha 1}$, $K_{\alpha 2}$; $K_{\beta 1}$, $K_{\beta 2}$ was discovered. When $\sqrt{\nu_\alpha}$ was plotted versus Z, even in this case, one got straight lines but with different slopes. So we can write

ν = C_n (Z – S)^2 where S is called the screening constant. Both the constants C_n and S are different for different series.

For hydrogen like atoms, we have

$$\nu = c\,R\,Z_{eff}^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\nu = c\,R\,(Z\text{-}S)^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where the effective nuclear charge has been replaced by (Z-S).

For the K series $n_i = 2$ and $n_f = 1$

Therefore $\nu_K = c R (Z\text{-}S)^2 (\frac{1}{1^2} - \frac{1}{2^2})$

$\nu_K = \frac{3}{4} c R (Z\text{-}S)^2$

When an electron jumps from the L shell to the K shell, the effective charge seen by the electron is $+Z_e$ on the nucleus plus (-e) on the K electron. Hence $Z_{eff} = (Z - 1)$

Hence above equation becomes $\nu_K = \frac{3}{4} c R (Z\text{-}1)^2$ which is Mosley's law as stated earlier.