Quadrant II - Notes

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Module Name: Magnetic Scalar Potential
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Notes

Magnetic Scalar Potential

$$\nabla \times \vec{B}(r_2) = \mu_o \vec{J}(r_2)$$

The above equation indicates that the curl of the magnetic induction is zero wherever the current density is zero. When this the case the magnetic induction in such regions can be considered as gradient

a scalar potential

$$\vec{B} = -\mu_o \nabla \varphi^*$$

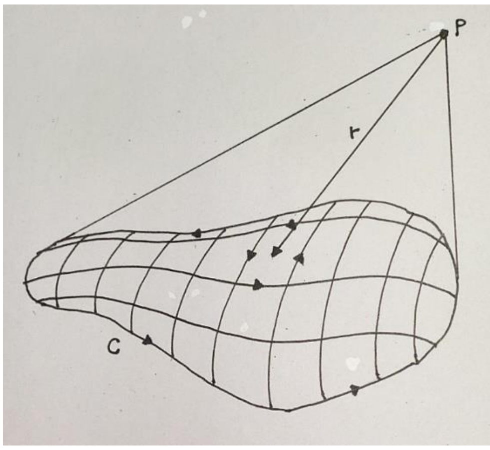
However the divergence of $\vec{B}$ is also zero

$$\nabla \times \vec{B} = -\mu_o \nabla \varphi^* = 0$$

$$\vec{B}(r_2) = \frac{\mu_o}{4\pi}\left[-\frac{\vec{m}}{r_2^3} + \frac{3(\vec{m}\cdot\vec{r}_2)\vec{r}_2}{r_2^5}\right]$$

$$= -\mu_o \nabla\left(\frac{\vec{m}\cdot\vec{r}_2}{4\pi r_2^3}\right)$$

$$\boldsymbol{\varphi}^* = \frac{\vec{\boldsymbol{m}} \cdot \vec{\boldsymbol{r}}_2}{4\pi \boldsymbol{r}_2^3} \qquad - \text{Eq(i)}$$



A large circuit $\boldsymbol{C}$ can be divided into many small circuits by means of a mesh, as shown in Figure If each small loop formed by a mesh carries the same current as originally was carried by the circuit $\boldsymbol{C}$, then, because of the cancellation of currents in the common branch of adjacent loops, the net effect is the same as if the charge flowed only in the circuit $\boldsymbol{C}$. For any one of the small loops, the magnetic moment may be written as

$$\boldsymbol{d\vec{m}} = \boldsymbol{I\vec{n}\, da} \qquad Eq(\text{ii})$$

Since each of the loops is sufficiently small to be regarded as planar. Using this expression in Eq. (i) and integrating over the surface bounded by $\boldsymbol{C}$ gives

$$\boldsymbol{\varphi} * (\boldsymbol{P}) = \frac{\boldsymbol{I}}{\boldsymbol{4\pi}} \int_{\boldsymbol{s}} \frac{\boldsymbol{r}_2 \cdot \boldsymbol{n\, da}}{\boldsymbol{r}_2^3} \qquad \text{Eq. (iii)}$$

In this equation $\boldsymbol{r}_2$ must be interpreted as the vector from $\boldsymbol{da}$ to the point $\boldsymbol{P}$, that is, $-\boldsymbol{r}$, is shown in Fig. 8-10. Making the change $\boldsymbol{r}_2 = -\boldsymbol{r}$ results in

$$\boldsymbol{\varphi} * (\boldsymbol{P}) = -\frac{\mathbf{1}}{\boldsymbol{4\pi}} \int_{\boldsymbol{s}} \frac{\boldsymbol{r} \cdot \boldsymbol{n\, da}}{\boldsymbol{r}^3} \qquad - \text{Eq. (iv)}$$

The quantity $\boldsymbol{r} \cdot \boldsymbol{n}\, \boldsymbol{da}$ is just $\boldsymbol{r}$ times the projection of $\boldsymbol{da}$ on a plane perpendicular to $\boldsymbol{r}$. Thus $\boldsymbol{r} \cdot \boldsymbol{n}\, \boldsymbol{da}/\boldsymbol{r}^3$ is the solid angle subtended by $\boldsymbol{da}$ at $\boldsymbol{P}$.

Eq. (iv) may be written as

$$\boldsymbol{\varphi} * (\boldsymbol{P}) = -\frac{\boldsymbol{I\Omega}}{\boldsymbol{4\pi}}$$

where $\boldsymbol{\Omega}$ is the solid angle subtended by the curve $\boldsymbol{C}$ at the point $\boldsymbol{P}$.

The magnetic scalar potential can be used for the calculation of the magnetic field either due to current-carrying circuits or to magnetic double layers (layers of dipoles). This procedure is occasionally useful in dealing with circuit problems; however, its principal use is in dealing with magnetic materials.
