Quadrant II – Transcript and Related Materials

Programme	**: Bachelor of Science (Third Year)**
Subject	**: Physics**
Course Code	**: PYC 110**
Course Title	**: Electromagnetic Theory II & Theory of Relativity**
Unit 12	**: Relativistic Dynamics**
Module Name	**: Longitudinal and Transverse Mass**
Module No	**: 16**
Name of the Presenter	**: Mr. Yatin P. Desai**

Notes

Mass in Relativity:

The relativistic expression for total energy of a particle is given by;

$E = c\sqrt{p^2 + m_0^2 c^2}$.

This expression is useful in high-energy physics to calculate the total energy of a particle when its momentum is given or vice versa.

By differentiating above equation with respect to p, we obtain another useful relation as follows;

$\frac{dE}{dp} = \frac{pc}{\sqrt{m_0^2 c^2 + p^2}}$

$\therefore \frac{dE}{dp} = \frac{pc^2}{c\sqrt{m_0^2 c^2 + p^2}} = \frac{pc^2}{E}$.

With $E = mc^2$ and $\vec{p} = m\vec{v}$, above equation reduces to; $\frac{dE}{dp} = v$

A force $\vec{F}$ is acting on a stationary particle having a rest mass m_0. The particle attains a velocity, $\vec{v}$. The corresponding mass m is given by;

$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

By Newton's 2nd law of motion;

$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

$\therefore \vec{F} = m\vec{a} + \vec{v}\frac{dm}{dt} ---------(1)$

Total energy of the particle is given by;

$E = m_0c^2 + K$

$mc^2 = m_0c^2 + K$

$\therefore m = m_0 + \frac{K}{c^2}$

Differentiating above equation;

$\frac{dm}{dt} = \frac{dm_0}{dt} + \frac{d}{dt}\left(\frac{K}{c^2}\right)$

$\therefore \frac{dm}{dt} = 0 + \frac{d}{dt}\left(\frac{K}{c^2}\right) ------------ (2)$

$dK = change\ in\ K.E.$

$= work\ done\ by\ external\ force\ on\ the\ particle$

$= dw$

$= \vec{F}.d\vec{r}$

Equation (2) can be written as;

$\frac{dm}{dt} = \frac{1}{c^2}\frac{\vec{F}.d\vec{r}}{dt} = \frac{1}{c^2}\vec{F}.\frac{d\vec{r}}{dt} = \frac{1}{c^2}(\vec{F}.\vec{v})$

$\therefore$ equation (1)becomes;

$\vec{F} = m\vec{a} + \frac{\vec{v}}{c^2}(\vec{F}.\vec{v})$

$\vec{F} = m\vec{a} + \frac{(\vec{F}.\vec{v})}{c^2}\vec{v}$

$\therefore m\vec{a} = \vec{F} - \frac{(\vec{F}.\vec{v})}{c^2}\vec{v}$

$\therefore \vec{a} = \frac{\vec{F}}{m} - \frac{(\vec{F}.\vec{v})}{mc^2}\vec{v} -------- (3)$

This equation shows that in relativistic mechanics, the acceleration $\vec{a}$ is not having the same direction as the force $\vec{F}$ because the second term is having the direction of velocity $\vec{v}$. [In classical mechanics, $\vec{F}$ and $\vec{a}$ have same direction.

There are two cases in which $\vec{a}$ would have the same direction as $\vec{F}$.

Case (i):

When a force $\vec{F}$ is parallel to velocity $\vec{v}$.

$$\vec{F}.\vec{v} = Fvcos0 = Fv$$

Therefore equation (3) becomes;

$$\vec{a} = \frac{\vec{F}}{m} - \frac{Fv}{mc^2}\vec{v}$$

Taking magnitudes on both sides;

$$a = \frac{F}{m}\left(1 - \frac{v^2}{c^2}\right)$$

$$\therefore F = \frac{ma}{1-\frac{v^2}{c^2}} = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} a$$

$$\therefore \vec{F} = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \vec{a}$$

Thus force $\vec{F}$ and hence acceleration $\vec{a}$ are parallel to the velocity $\vec{v}$ *i.e.* parallel to the direction of motion. Hence above equation can be written as;

$$\therefore \vec{F}_{\parallel} = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \vec{a}_{\parallel}$$

Quantity $\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$; is the mass experienced when force acts in the direction parallel to the direction of motion. It is called as **longitudinal mass**.

Case (ii):

When force $\vec{F}$ is perpendicular to the velocity $\vec{v}$.

$$\vec{F}.\vec{v} = Fvcos90 = 0$$

Therefore equation (3) becomes;

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\therefore F = ma = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} a$$

Thus force $\overrightarrow{F}$ and hence acceleration $\overrightarrow{a}$ are perpendicular to the velocity $\overrightarrow{v}$ *i.e.* perpendicular to the direction of motion. Hence above equation can be written as;

$$\therefore \overrightarrow{\mathrm{F}}_{\perp} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\overrightarrow{\mathrm{a}}_{\perp}$$

The quantity $\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ is the mass experienced when the force acts in the direction perpendicular to the direction of motion. It is called as **transverse mass.**