Quadrant II – Glossary

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Unit 1: Nuclear Properties

Module Name: Magnetic and electrical dipole moments

Module No: 03

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Magnetic dipole moments

Particle having a charge q and mass m circulates with speed v in a circular orbit of radius r. If it has a time period t, then the current i associated with the charge e is

$$i = e/t \text{ , t can be calculated as } t = 2\pi r/ v$$

$$t = e\ v/ 2\pi r ,$$

Now magnetic dipole moment μ for current i around an area of the loop A is given as

$$\mu = i\ \pi r^2$$

$$\mu = (e\ v/ 2\pi r)\ \pi r^2$$

$$\mu = e v r /2$$

$$\text{Angular momentum } L = m v r$$

$$\mu = (e/2m)L$$

Magnetic dipole moment of the nucleus occurs due to the motion of charged particles.

A moving charge has an orbital magnetic dipole moment which is proportional to its orbital angular momentum L and is given by

$$\mu_l = \left(\frac{e}{2m}\right)L$$

For proton $m = m_p$

$$\mu_l = \left(\frac{e}{2m_p}\right)L$$

Neutrons have no charge also they do not have an orbital magnetic dipole moment. The magnetic moment along the Z-axis is $\mu_{l,z} = \left(\frac{e}{2mp}\right)L_z$

$$\mu_{l,z} = \left(\frac{e\hbar}{2mp}\right)ml$$

$$\mu_{l,z} = ¿\ \mu_N ml$$

We define a new constant $\mu_N = \frac{e\hbar}{2mp}$ called as nuclear magneton

If the particle has Spin S,

Magnetic moment may have an additional Spin magnetic dipole moment

$$\mu_s = g_s\left(\frac{e}{2mp}\right)s$$

$$\mu_{s,z} = g_s \mu_N M_s$$

g_s is the spin gyromagnetic ratio

The resultant magnetic dipole moment is the vector sum of the magnetic dipole moments of all the nucleons.

Given by $\mu \¿ g_l \left(\frac{e}{2mp} \right) L$

$$\mu_z \¿ g_l \left(\frac{e}{2mp} \right) L_z$$

Since $L_z = M_l \hbar$

$$\mu_z \¿ \mu_N g_l M_l$$

M_l is substituted by l,

$$\mu_z \¿ \mu_N g_l l$$

$$\mu = \frac{\mu_z}{\mu_N} \¿ g_l l$$

g_l is the nuclear gyromagnetic ratio

Electric quadruple moment

This property arises because of non-spherical charge distribution in the nucleus. It is very important to note that for a nucleus having spherical shape, neither dipole moment exists nor quadrupole moment exists. Because the center of mass of the nucleus coincides with center of charge of the nucleus.

For non-spherical nuclei, It is assume that center of mass is at the origin and center of charge is not at the origin. The potential at any point *P* located outside the nucleus at a distance *r* from the origin is given by

$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{e}{\¿ r - r' \vee \¿ \¿}$ ..... (1)

e is charge placed at a distance *r* from the origin

$$|r\text{-}r'| = [r2+(r')2-2rr'\cos\theta]^{1/2}$$

θ is the angle between r-r’ , Substitute the |r-r’| in equation (1).

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{e}{¿[r2+(r')2-2rr'\cos\theta]^{1/2}}$$

$$V(r) = \frac{e}{4\pi\varepsilon_0 r}\left(1+\frac{(r')2}{r2}-\frac{2r'}{r}\cos\theta\right)^{-1/2} \text{.....(2)}$$

$$r' << r$$

$$\left(1+\frac{(r')2}{r2}-\frac{2r'}{r}\cos\theta\right)^{-1/2} = 1+\frac{r'}{r}\cos\theta+\left(\frac{r'}{r}\right)^2\left(\frac{3}{2}\cos2\theta-\frac{1}{2}\right)$$

$$+\left(\frac{r'}{r}\right)^3\left(\frac{5}{2}\cos3\theta-\frac{3}{2}\cos\theta\right)+....$$

Substitute in equation (2)

$$V = \frac{e}{4\pi\varepsilon_0 r}¿+\left(\frac{r'}{r}\right)^3\left(\frac{5}{2}\cos3\theta-\frac{3}{2}\cos\theta\right)+....]$$

$$V = \frac{e}{4\pi\varepsilon_0 r} + \frac{er'}{4\pi\varepsilon_0 r2}\cos\theta+ \frac{e(r')2}{4\pi\varepsilon_0 r3}\left(\frac{3}{2}\cos2\theta-\frac{1}{2}\right)+...$$

$$V = \frac{e}{4\pi\varepsilon_0 r} + \frac{er'}{4\pi\varepsilon_0 r2}\cos\theta+ \frac{e(r')2}{4\pi\varepsilon_0 r3}\left(\frac{3}{2}\cos2\theta-\frac{1}{2}\right)+...$$

First term on the right-hand side represents the contribution to potential from single charge.

The second term represents the contribution due to a dipole.

Third term represents the contribution due to electric quadrupole

whose quadrupole moment is given as

$$Q = e(r')2\left(\frac{3}{2}\cos2\theta-\frac{1}{2}\right)$$

The dipole the moment of the nucleus is zero.

If $Q > 0$, then nucleus is prolate in shape,

If $Q < 0$, then nucleus is oblate in shape

If $Q = 0$, then nucleus is spherical in shape