Quadrant II – Notes

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Nuclear Force

Perhaps more man hours have been devoted to the study of nuclear foerces than to any other scientific question in history. The existence of a nuclei can be explained only if a new type of force in nature is assumed. The nuclear force can be studied by studying scattering of the (n-p), (p-p) and (n-n) systems. The (n-p) system can be studied by using a beam of neutrons from a reactor and scattering them off a hydrogenous target. The only bound state of the (n-p) system found in nature is the deuteron. The dineutron (n-n) and diproton (p-p) do not exist in the bound state.

Main Characteristics of Nuclear Forces:

- The nuclear force is strongly attractive. At short distances it is stronger than the Coulomb force.
- The nuclear force is short ranged (10^{-14} m). This can be inferred from the B.E/A curve being approximately constant.
- The nuclear force is charge independent. The nucleon –nucleon force seems to be independent whether the nucleon are neutrons or protons.
- The nuclear force is spin dependent. The nucleon –nucleon force seems to be dependent whether the spins of the nucleons are parallel or antiparallel.
- The nuclear force has a repulsive core. The nucleon-nucleon force includes a repulsive term, which keeps the nucleons at a certain average separation.

- The nuclear force is not completely central. The nucleon-nucleon force has a non-central or tensor component. This part of the force does not conserve orbital angular momentum

Properties of the Deuteron

The only bound system of two nucleons found in nature is the deuteron, which consists of a neutron and a proton.

- **Binding energy of a deuteron** = 2.2256 MeV
- **Average radius of deuteron** = 4.2 Fermi
- **Spin**

  The measured total angular momentum (or spin) for the deuteron is $I = 1$. The total angular momentum I of the deuteron should have three components: the orbital angular momentum l of the nucleons as they move about their common centre of mass and the individual spins s_n and s_p of the neutron and proton.

$$I = s_n + s_p + l = l + S$$

  where S is the total spin quantum number and can take the values, $S = |s_n - s_p|, .., |s_n + s_p|$ in steps of 1.

  Since the spins of both neutron and protons are ½, then

$$S = 1 \text{ or } 0$$

  As the measured value of $I = |S - l|, \ldots, |S + l| = 1,$ the orbital angular momentum can take different values, given as

  If $S = 1$ and $l = 0$ then $I = 1$

  If $S = 1$ and $l = 1$ then $I = 0,\ 1,\ 2$

  If $S = 1$ and $l = 2$ then $I = 1, 2, 3$

  If $S = 0$ and $l = 1$ then $I = 1$

  The parity associated with orbital motion is $(-1)^l$. Hence, for $l = 0$ (s states) and $l = 2$ (d states) parity is even and for $l = 1$ (p states) parity is odd. By studying reactions of deuterons and property of photon emitted during formations deuterons we know parity of deuteron is even. Therefore, we can exclude the combination of spins than include $l = 1$ and leaving $l = 0$ and $l = 2$ as only possibilities.

- **Magnetic Dipole Moment:**

If the deuteron is in an $l = 0$ state, there should be no orbital contribution to the magnetic moment and we can assume the total magnetic moment to be combination of neutron and proton magnetic moments

$$\boldsymbol{\mu} = \boldsymbol{\mu_n} + \boldsymbol{\mu_p}$$

$$\boldsymbol{\mu} = \frac{g_{sn}\mu_N}{\hbar}\boldsymbol{s}_n + \frac{g_{sp}\mu_N}{\hbar}\boldsymbol{s}_p$$

where $\mu_N(\text{nuclear magneton}) = \frac{e\hbar}{2M_p} = 3.1525 \times 10^{-8}\frac{eV}{T},\ g_{sn} = 3.826084, \text{and} \ \ g_{sp} = 5.5859.$

Hence, the expected value of magnetic moment is $\mu = 0.879804\ \mu_N$

The observed value of the magnetic moment is given as

$$\mu_{obs} = 0.8574376 \pm 0.0000004\ \mu_N$$

The difference between the between the observed and expected value $\mu = 0.879804\ \mu_N$ although not large but still needs to be explained. The discrepancy could arise from a small mixture of d state ($l = 2$) in the deuteron wave function.

$$\psi = a_0\psi(l = 0) + a_2\psi(l = 2)$$

The system spends a fraction $a_0^*a_0 = |a_0|^2$ of its time in an $l = 0$ state and a fraction $|a_2|^2$ of its time in $l = 2$ state. Calculating the magnetic moment from the wave function gives

$$\mu = |a_0|^2\mu(l = 0) + |a_2|^2\mu(l = 2)$$

The observed value is consistent with $|a_0|^2 = 0.96$ and $|a_2|^2 = 0.04$. That is the deuteron is in $l = 0$ state about 96% of the time and is in $l = 2$ state only 4% of the time. Thus, the observed magnetic dipole can be accounted for by considering a mixture of $l = 2$ in the deuteron wave function.

- **Quadrupole Moment**

If the deuteron is in $l = 0$ state the wave function has no $(\theta,\ \phi)$ dependence and hence is a function of only r. The wave function must therefore exhibit spherical symmetry implying a zero quadrupole moment. The observed quadrupole moment is

$$Q = 0.00288 \pm 0.00002\ \text{b}$$

where, 1 b (barn)=$10^{-28}m^2$.

The results of the electric quadrupole moment can be explained with $|a_0|^2 = 0.96$ and $|a_2|^2 = 0.04$. Thus, the assumption of the pure $l = 0$ state is not quite exact

Tensor Force:

The conservation of orbital angular momentum is violated as the wave function contains a mixture of l values. From elementary mechanics, just as linear momentum can be only changed by a force, the angular momentum can be changed only by a torque. The torque acting on the body is

$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = rF_\theta = -\frac{\partial V}{\partial \theta}$$

A changing angular momentum implies that the potential $V$is a function of $\theta$and not only of r. Since a central force is defined as the one for which V is only a function of r, this is a non-central force also called as a tensor force.

The angle θ upon which the tensor force depends must be measured from the direction of the spin vector $\vec{S}$. When the spin direction is perpendicular to the line joining the nucleons, the force is repulsive and when the spin direction is parallel to the line joining the nucleons the force is attractive.

Illustration of the tensor force. The force is attractive or repulsive depends on the way the magnets are oriented.