Quadrant II

Programme: Bachelor of Science (Third Year)

Subject: Physics

Course Code: PYD106

Course Title: Nuclear Physics

Unit 4: Nuclear reactions

Module Name: Energetics of nuclear reactions, Q value

Module No: 15

Q-value of Nuclear Reactions:

- The important part of nuclear reactions is the energy released or taken-up in the reaction.
- It is called as **nuclear reaction energy** and is denoted by **Q-Value**.
- It also represents the difference between kinetic energies of products of the reaction and that of incident particles.

Q-value in Nuclear Reaction

Let us consider a reaction $x + X \longrightarrow Y + y$

x – incident particle (KE = E_x)

X – the target particle (KE = 0)

Y – the product nuclei (KE = E_Y)

y – the product particle (KE = E_y)

From Law of conservation of energy we have

$$(E_x + m_x c^2) + M_X c^2 = (M_Y c^2 + E_Y) + (m_y c^2 + E_y) \quad -(1)$$

The Q-value of the reaction is

$$Q = E_Y + E_y - E_x$$

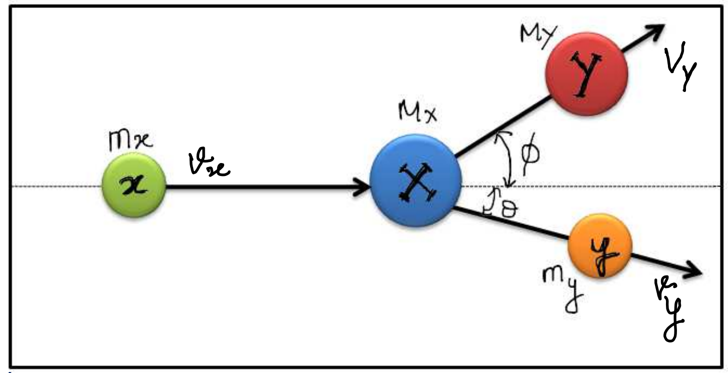
$$\therefore \boxed{Q = (M_X + m_x)c^2 - (M_Y + m_y)c^2}$$

$Q > 0$ exothermic

$Q < 0$ endothermic

Q-value in Nuclear Reaction

Consider a reaction $x + X \rightarrow Y + y + Q$



Applying law of conservation of linear momentum along x-axis & y-axis we have

$$m_x v_x = M_y V_y \cos\phi + m_y v_y \cos\theta \quad — (1)$$

$$0 = M_y V_y \sin\phi - m_y v_y \sin\theta \quad — (2)$$

From (1) $m_x v_x - m_y v_y \cos\theta = M_y V_y \cos\phi \quad — (3)$

From (2) $M_y V_y \sin\phi = m_y v_y \sin\theta \quad — (4)$

Squaring (3) and (4) and adding them we get

$$(m_x v_x)^2 + (m_y v_y)^2 - 2 m_x v_x m_y v_y \cos\theta = (M_y V_y)^2 \quad — (5)$$

Q-value in Nuclear Reaction

We have $E_x = \frac{1}{2} m_x v_x^2$, $E_y = \frac{1}{2} m_y v_y^2$ and $E_Y = \frac{1}{2} M_Y V_Y^2$

$$2E_x m_x = (m_x v_x)^2, \quad 2E_y m_y = (m_y v_y)^2$$

$\therefore$ Equation (5) becomes

$$2E_x m_x + 2E_y m_y - 4\sqrt{E_x m_x E_y m_y}\cos\theta = 2E_Y M_Y$$

$$\therefore E_Y = \frac{E_x m_x}{M_Y} + \frac{E_y m_y}{M_Y} - \frac{2}{M_Y}\sqrt{E_x m_x E_y m_y}\cos\theta \quad \text{--- (6)}$$

$\therefore$ The Q-value of the reaction can be written as $Q = E_Y + E_y - E_x$

$$\therefore Q = E_x\left(\frac{m_x}{M_Y} - 1\right) + E_y\left(\frac{m_y}{M_Y} + 1\right) - \frac{2}{M_Y}\sqrt{E_x m_x E_y m_y}\cos\theta \quad \text{--- (7)}$$