Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (Third year)

Subject: Physics

Course Code: PYD 106

Course Title: Nuclear Physics

Unit: 6A - Nuclear Models: Liquid drop model.

Module Name: Mass parabolas: Prediction of stability against decay for members of an isobaric family, spontaneous and induced fission.

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Notes:

❖ **Mass parabolas and prediction of stability against beta activity**

The Wiezsacker's semi empirical mass formula helps to explain the beta activity and the stability properties of isobars.

$$ {}^{A}_{Z}M = ZM_p + (A - Z)M_n - a_v A + a_s A^{\frac{2}{3}} + a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} + a_a \frac{(A-2Z)^2}{A} \pm E_p $$

(in a.m.u)

$$ M(Z, A) = \alpha A + \beta Z + \gamma Z^2 \pm \delta \ldots (A) $$

The simplified expression for the Wiezsacker's semi empirical mass formula

$$ \alpha = M_n - \left(a_v - a_a - \frac{a_s}{A^{\frac{1}{3}}} \right) \quad ; $$

$$ \beta = -4a_a - (M_n - M_p) \quad ; $$

$$ \gamma = \left(\frac{4a_a}{A} + \frac{a_c}{A^{\frac{1}{3}}} \right) $$

$\pm\delta$ is the pairing energy

pairing energy $= +\,\delta$ for odd Z , odd N

$\text{pairing energy} = 0 \text{ for odd Z , even N}$

$\text{pairing energy} = 0 \text{ for even Z , odd N}$

$\text{pairing energy} = -\,\delta \text{ for even Z , even N}$

Equation (A) is the simplified expression for the Wiezsacker's semi- empirical mass formula.

A family of different nuclides for which the mass number A is the same is called an isobaric family.

When A in equation (A) is a constant, it's the equation of a parabola.

Thus, the plots of M against Z are parabolic in shape with a minimum corresponding to that value of Z which gives the most stable isobar in the isobaric family.

For odd A nuclei, $\delta = 0$ and thus there is only one parabola implying that there is only one stable state.

For even A nuclei there can be up to three stable nuclei.

- **Odd A isobar**

The relationship between the masses M(Z,A) of the members of the isobaric family is given by the equation:

$\mathrm{M(Z, A)} = \alpha\mathrm{A} + \beta\mathrm{Z} + \gamma\mathrm{Z}^2 \pm \delta \ldots \mathrm{(A)}$

For any odd A nuclide, the correction for pairing energy term $\delta = 0$

Thus, for any group of odd –A isobars the relationship between the masses M and the charge Z is given by:

$$\mathrm{M(Z, A)} = \alpha\mathrm{A} + \beta\mathrm{Z} + \gamma\mathrm{Z}^2 \ldots (1)$$

Equation (A) is a quadratic in Z and so for each value of A, there is a specific value of Z for which ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{M}$ is minimum.

- Charge of the most stable odd A isobar

The Z value which corresponds to minimum mass M is called the nuclear charge of the most stable isobar, denoted by $\mathrm{Z_o}$

$$\left(\frac{\partial \mathrm{M}}{\partial \mathrm{Z}}\right)_{\mathrm{A=constant}} = 0$$

$$\mathrm{M(Z, A)} = \alpha \mathrm{A} + \beta \mathrm{Z} + \gamma \mathrm{Z}^2 \ldots (1)$$

$$\left(\frac{\partial \mathrm{M}}{\partial \mathrm{Z}}\right)_{\mathrm{A=constant}} = \beta + 2\gamma \mathrm{Z_o} = 0 \ldots (2)$$

$$\therefore \mathrm{Z_o} = -\frac{\beta}{2\gamma} \ldots (3)$$

This is the charge of the most stable isobar.

- **Mass of the most stable odd-A isobar**

Charge of the most stable isobar

$$\mathrm{Z_o} = -\frac{\beta}{2\gamma} \ldots (3)$$

Substitute the value of β from the equation for $\mathrm{Z_o}$ in equation (1) to obtain an expression for the mass of the most stable odd A isobar.

$$\mathrm{M(Z, A)} = \alpha \mathrm{A} + \beta \mathrm{Z} + \gamma \mathrm{Z}^2 \ldots (1)$$

$$\mathrm{M(Z_o, A)} = \alpha \mathrm{A} + \beta \mathrm{Z_o} + \gamma \mathrm{Z_o}^2$$

$$\mathrm{M(Z_o, A)} = \alpha \mathrm{A} - 2\gamma \mathrm{Z_o}^2 + \gamma \mathrm{Z_o}^2$$

$$\mathrm{M(Z_o, A)} = \alpha \mathrm{A} - \gamma \mathrm{Z_o}^2 \ldots (4)$$

This is the mass of the most stable isobar.

- **Parabolic mass relationship for odd –A isobars**

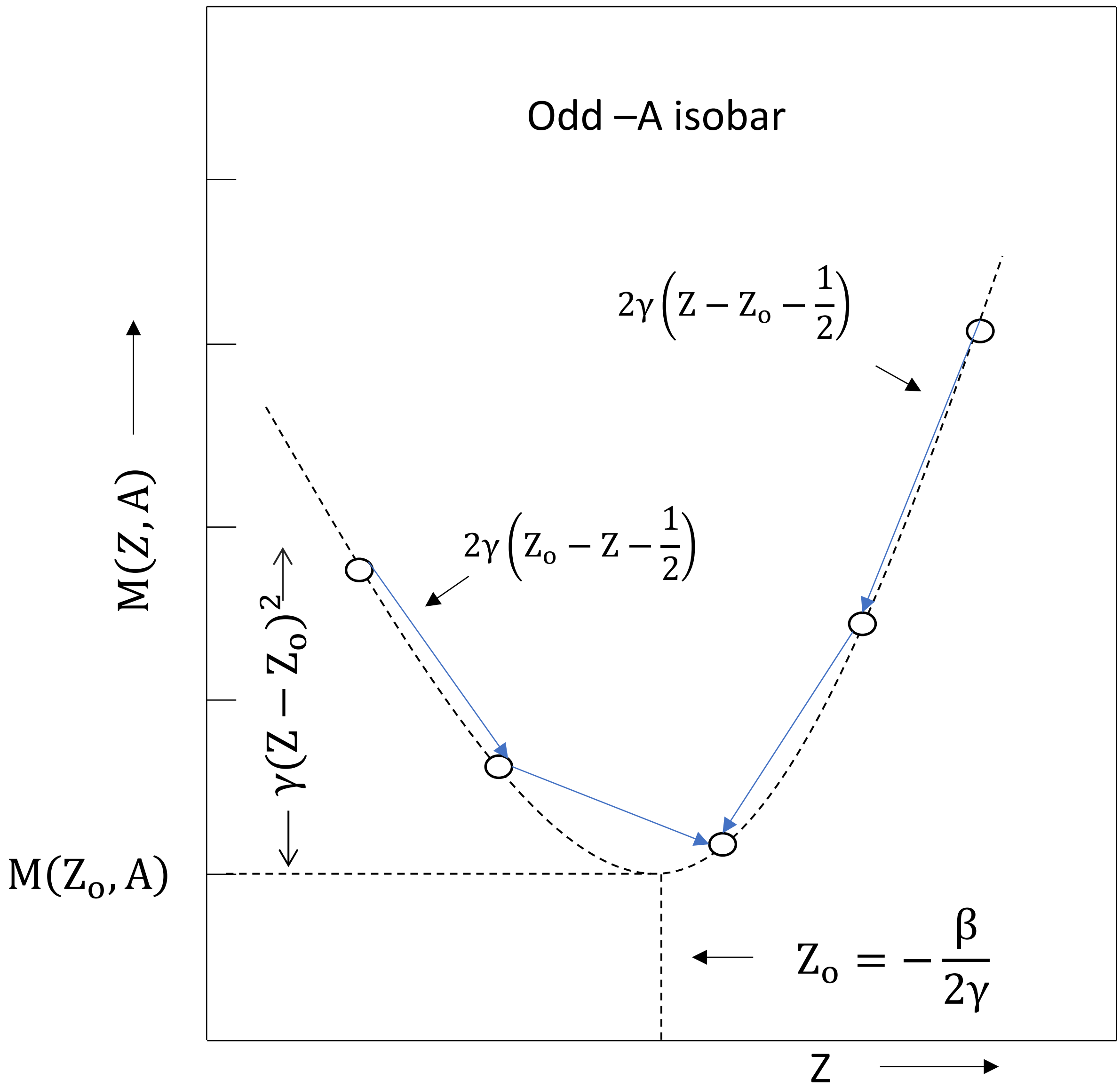
The difference in masses for odd A can be obtained by subtracting equation (4) from equation (2)

The difference in masses for odd A is: $= \mathrm{M(Z, A)} - \mathrm{M(Z_o, A)}$

$$= \alpha \mathrm{A} + \beta \mathrm{Z} + \gamma \mathrm{Z}^2 - \alpha \mathrm{A} - \gamma \mathrm{Z_o}^2$$

$$\mathrm{M(Z, A)} - \mathrm{M(Z_o, A)} = \gamma(\mathrm{Z} - \mathrm{Z_o})^2 \ldots (5)$$

By making use of equations for α and γ, it is possible to estimate $\mathrm{M(Z_o, A)}$ and $\mathrm{M(Z, A)} - \mathrm{M(Z_o, A)}$.



The figure shows the parabolic mass relationship for odd A isobar with vertex at Z_o, $M(Z_o, A)$

There is one stable odd A isobar at the bottom of the curve with the greatest binding energy.

The isobars having masses greater than that of the stable isobar decay by β^- decay leading to an increase in Z or decay by β^+ decay leading to decrease in Z to the most stable isobar.

For the transitions between the odd-A isobars, the Q-values can be obtained as follows :

For β^- decay , the energy released,

i.e the Q-value is given by : $Q_{\beta^-} = M(Z, A) - M(Z + 1, A)$

$$Q_{\beta^-} = \gamma[(Z - Z_o)^2 - (Z + 1 - Z_o)^2] \ldots \text{(from equation 5)}$$

$$Q_{\beta^-} = 2\gamma\left(Z_o - Z - \frac{1}{2}\right)$$

Similarly, for β^+ decay , the energy released is $Q_{\beta^+} = 2\gamma\left(Z - Z_o - \frac{1}{2}\right)$

- **Even A nuclei**

The relationship between the masses M(Z,A) of the members of the isobaric family is given by the equation:

$$M(Z, A) = \alpha A + \beta Z + \gamma Z^2 \pm \delta \ldots (A)$$

The isobaric masses for even A nuclides follow the same pattern as for odd A except for the paring energy.

For any Even A nuclide , the charge of the most stable isobar

$$Z_o = -\frac{\beta}{2\gamma}$$

The mass of the most stable even A isobar is

$$M(Z_o, A) = \alpha A + \gamma {Z_o}^2 - \delta \ldots (i)$$

The negative pairing energy is taken so that $M(Z_o, A)$ will have the smallest possible value

The difference in masses for odd A can be obtained by subtracting equation(i)from equation(A). Thus, the difference in masses for Even A is:

$$M(Z, A) - M(Z_o, A) = \alpha A + \beta Z + \gamma Z^2 \pm \delta - \alpha A + \gamma {Z_o}^2 + \delta$$

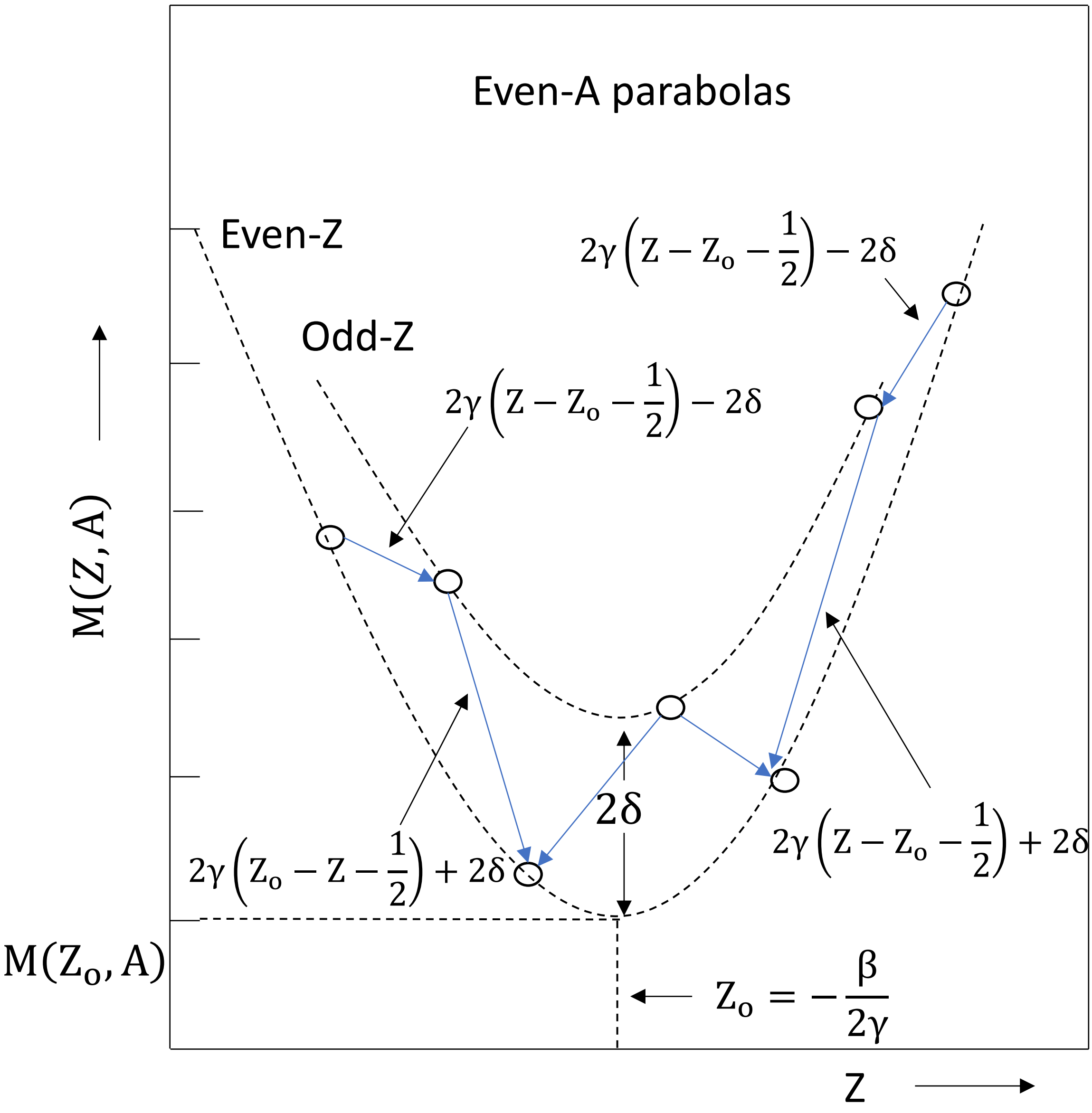
$$M(Z, A) - M(Z_o, A) = -2\gamma Z Z_o + \gamma Z^2 + \gamma {Z_o}^2 \pm \delta + \delta$$

$$M(Z, A) - M(Z_o, A) = -\gamma(Z - Z_o)^2 + \begin{Bmatrix} 2\delta \text{ for odd } Z \\ 0 \text{ for even } Z \end{Bmatrix} \ldots (6)$$

The pairing energy correction to the mass is taken into account.

its 2δ for odd Z and zero for even nuclides.

Equation (6) gives the parabolic mass relationship for even A isobars.



There are two curves one for even Z- even N nuclides and another for odd Z –odd N nuclides.

The odd Z nuclides lie on the upper curve and are therefore unstable with respect to those on the lower curve. The only exceptions to this rule are the light nuclides H^2, Li^6, B^{10} and N^{14}.

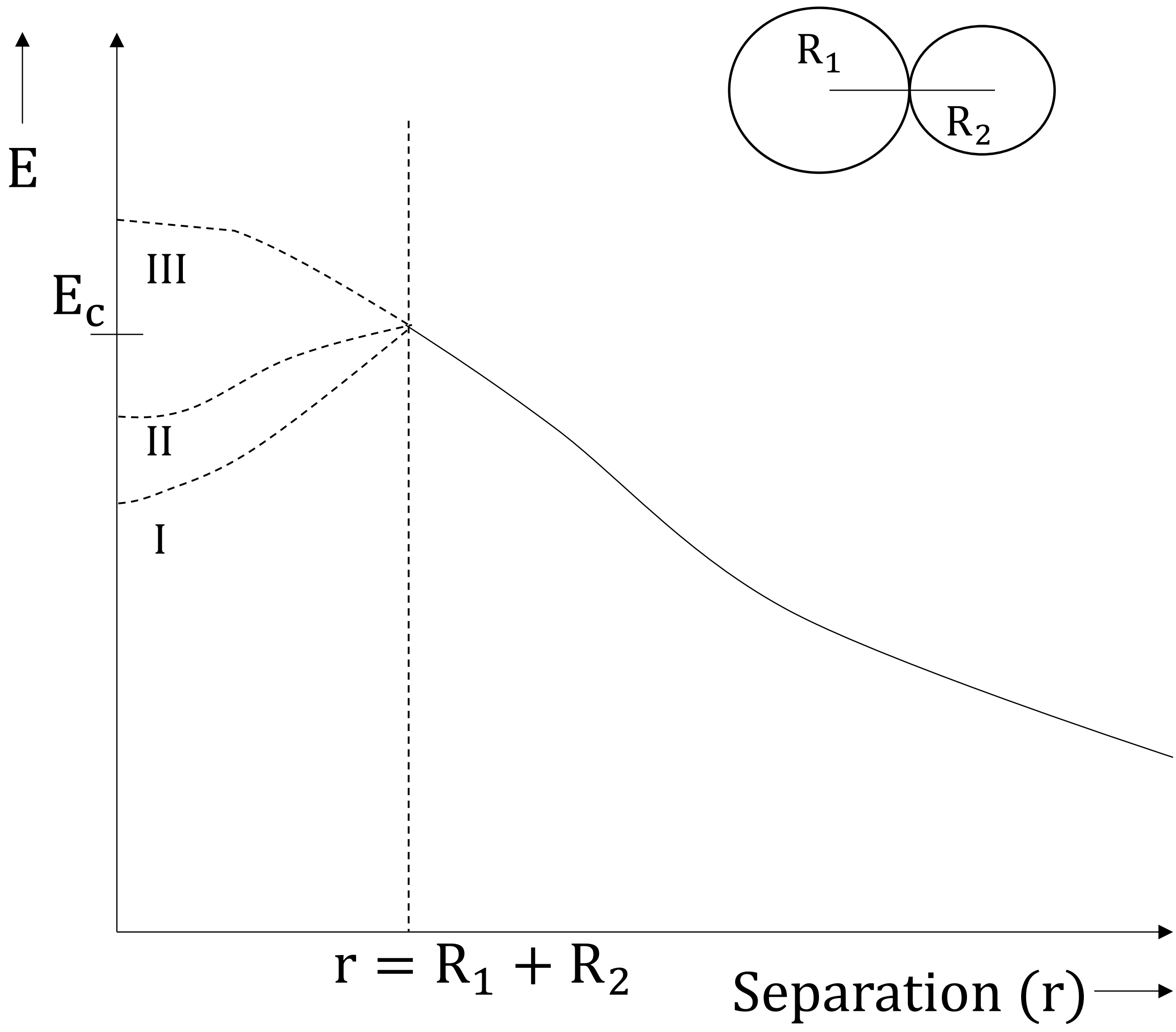
Nuclides to the left of the stable nuclides decay by β^- emission or K-capture or both and those to the right by β^+ emission or K-capture or both.

The isobar at the bottom of the odd Z curve may decay either by β^+ or β^- emission .

The Q values for the decay process can be obtained in a manner similar to Odd- A isobars.

❖ Spontaneous fission and induced fission

- A nucleus' spherical shape depends on the balance between the surface energy forces and the Coulomb repulsive forces.
- If excitation energy is provided to the nucleus, the spherical shape of nucleus tends to distorts to an ellipsoidal shape. If the surface energy forces tend to restore the nucleus back to the original shape.
- When the excitation energy is sufficiently large the nucleus undergoes fission.
- The surface area and hence the surface energy is higher after fission, but the Coulomb energy is lower. Thus, fission is accompanied by a net drop in potential energy.



The Potential energy E is plotted against a parameter r which is a measure of degree of deformation.

At $\mathrm{r} = 0$, the spherical nucleus has energy $\mathrm{E_o}$.

When the two fragments just touch at $\mathrm{r} = \mathrm{R_1} + \mathrm{R_2}$, the coulomb energy is denoted by $\mathrm{E_c}$.

$$\mathrm{E_c} = \frac{\mathrm{Z_1 Z_2 e^2}}{\mathrm{R_1 + R_2}}$$

The energy at $\mathrm{r} = \infty$ is taken equal to zero

For, $\mathrm{r} > \mathrm{R_1} + \mathrm{R_2}$, the energy is the electrostatic energy due to the mutual repulsion between the two positively charged fragments.

For, $\mathrm{r} < \mathrm{R_1} + \mathrm{R_2}$, the energy depends not only on the electrostatic forces but also on the surface energy.

The values of E in $\mathrm{r} < \mathrm{R_1} + \mathrm{R_2}$ region is shown by the Three different curves (I,II, III).

The shapes of these curves and the values of $\mathrm{E_o}$ are related to the mass of the nucleus, that is to the value of A.

Stable nuclei with values of A somewhat greater than 100 are of type I with $\mathrm{E_o}$ about 50 MeV smaller than $\mathrm{E_c}$.

Nuclei like those of uranium, plutonium or thorium are of type II.

For still heaver nuclei, $\mathrm{E_o}$ may be greater than $\mathrm{E_c}$ as in case of type III.

Nuclei of type III should undergo fission spontaneously and would not be expected to last long.

The activation energy $\mathrm{E_c} - \mathrm{E_o}$ needed to induce fission in a nucleus of the type II can be calculated from the Bohr Wheeler theory.