Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (Third Year)

Subject: Mathematics

Course Code: MTC 109

Course Title: Complex Analysis

Unit: Unit 3: Elementary Functions

Module Name: Hyperbolic Functions

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Notes:

Hyperbolic Functions
Definition:
The complex hyperbolic functions $cosh, sinh$: $\mathcal{C} \longrightarrow \mathcal{C}$ are defined by

$$Sinh\, z = \frac{e^z - e^{-z}}{2} \qquad Cosh\, z = \frac{e^z + e^{-z}}{2}$$

Properties /Identities:
1.$Sinh$,$Cosh$: $\mathcal{C} \longrightarrow \mathcal{C}$ are **entire** functions and
(i) $\frac{d}{dz}(\text{Sinh}\, z) = \text{Cosh}\, z$
(ii) $\frac{d}{dz}(\text{Cosh}\, z) = \text{Sinh}\, z$
2.The functions $\text{Sinh}\, z$ and $\text{Cosh}\, z$ are **periodic** with period $2\pi i$
3. $Sinh\,(-z) = -Sinh\,(z)$;
$Cosh\,(-z) = Cosh\,(z)$
4. $cosh^2 z - sinh^2 z = 1 \quad \forall\, z \in \mathcal{C}$
5.Relation between Hyperbolic and Trignometric functions

$$\text{Cosh}(iz) = \text{Cos}\, z \quad ; \quad Sinh(iz) = iSin\, z$$
$$\text{Cos}(iz) = \text{Cosh}\, z \quad ; \quad Sin(iz) = iSinh\, z$$

6. For complex numbers z_1 and z_2
Sinh $(z_1 + z_2)$ = Sinh z_1Cosh z_2+ Cosh z_1Sinh z_2
Cosh $(z_1 + z_2)$ = Cosh z_1Cosh z_2+ Sinh z_1Sinh z_2
7. $Sinh\, z = Sinh\, x\, Cosy + i\, Coshx\, Siny$
$Cosh\, z = Cosh\, x\, Cosy + i\, Sinh\, x\, Siny$

8. $|sinhz|^2 = sinh^2 x + sin^2 y$

$$|coshz|^2 = sinh^2 x + cos^2 y$$

Zeros of Hyperbolic Functions

9. The zeroes of $Sinh\, z$ are purely imaginary

i.e. $Sinh\, z = 0 \Rightarrow z = n\pi i$

10. The zeroes of $Cosh\, z$ are purely imaginary

$$Cosh\, z = 0 \Rightarrow z = (2n+1)\frac{\pi i}{2}$$

Other Hyperbolic Functions

$Tanh\, z = \frac{\mathrm{Sinh}\, z}{\mathrm{Cosh}\, z}$ $\qquad$ $Coth\, z = \frac{\mathrm{Coth}\, z}{\mathrm{Sinh}\, z}$

$Sech\, z = \frac{1}{\cosh z}$ $\qquad$ $Cosech\, z = \frac{1}{\sinh z}$

Singularities and Derivatives of
$\boldsymbol{Tanh\, z, Sech\, z, Coth\, z, \ \ Cosech\, z}$

The singularities of $Tanh\, z$ and $Sech\, z$ are the zeros of
$Cosh\, z$ i.e. $z = (2n+1)\frac{\pi i}{2}$
Except at these points $Tanh\, z$ and $Sech\, z$ are analytic everywhere and their derivatives are

$$\frac{d}{dz}(\mathrm{Tan}h\, z) = sech^2\, z \quad ; \frac{d}{dz}(\mathrm{Sech}\, z) = \mathrm{Sech}\, z \ Tanh\, z$$

Similarly, The singularities of $Coth\, z$ and $Cosech\, z$ are the zeros of $Sinh\, z$ i.e. $z = n\pi i$
Except these points $Coth\, z$ and $Cosech\, z$ are analytic everywhere and their derivatives are

$$\frac{d}{dz}(\mathrm{Cot}h\, z) = -Cosech^2\, z \quad ;$$

$$\frac{d}{dz}(\mathrm{Cosec}h\, z) = -Cosech\, z\, Coth\, z$$

Periodiciy of $\boldsymbol{Tanh\, z}$ and $\boldsymbol{Coth\, z}$

$Tanh\, z$ and $Coth\, z$ are **periodic** with period πi
since $Tanh\,(z + \pi i) = Tanh\, z$ and $Coth\,(z + \pi i) = Coth\, z$