Quadrant II - Notes

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Unit 6: X-rays

Module Name: Bragg's law, Bragg single crystal spectrometer
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Bragg's Law :

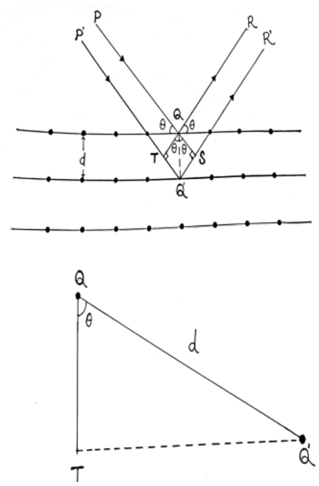
W.H.Bragg & W.L.Bragg discovered that diffraction of X-rays by solids could be treated as reflection from evenly spaced planes if monochromatic x-rays were used. For this discovery they got Nobel Prize in 1915. By using this law one can determine wavelength λ if inter planar spacing d is known.

Statement :

When monochromatic X-rays are made incident upon the atoms in a crystal lattice, each atom acts as a source of scattering radiation of the same wavelength. The crystal acts as a series of parallel reflecting planes. Constructive interference occurs when the path difference between the reflected X-rays is integer multiple of wavelength.

Bragg's Condition

$$n\lambda = 2d \sin\theta$$



Path-difference between the two rays, $TQ' + Q'S = d \sin\theta + d \sin\theta$

$= 2d \sin\theta.$

Hence the two rays will have constructive interference and produce maximum intensity, if

$2d \sin\theta = n\lambda$

$$\mathbf{2d \sin\theta = n\lambda}$$

Where n = 1, 2, 3...... The integer n gives the order of the scattered beam, λ is the wavelength of the X-rays used.

<u>The Bragg X-ray Spectrometer</u>

The essential parts of a Bragg spectrometer are shown in the fig(1) It is similar in construction to an optical spectrometer.

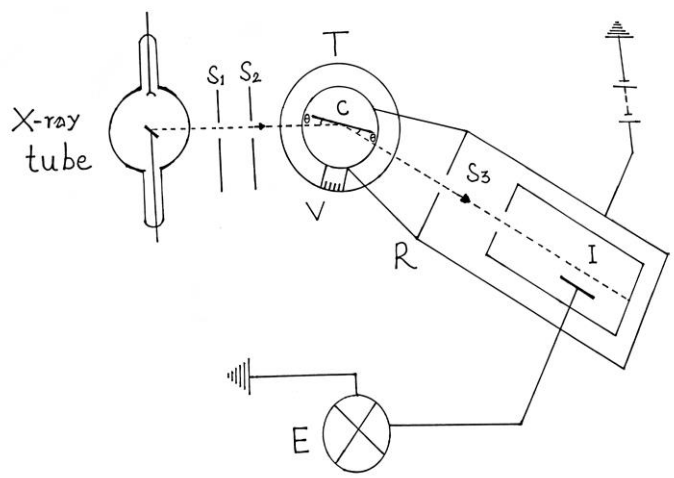


Fig.1

It consists of three parts.

(1) a source of X-rays

(2) a crystal held on a circular table which is graduated and provided with vernier

(3) a detector (ionisation chamber).

X-rays from an X-ray tube, limited by two narrow lead slits S_1 and S_2, are allowed to fall upon the crystal C. The crystal is mounted on the circular table T, which can rotate about a vertical axis and its position can be determined by the Vernier. The table is provided with radial arm (R) which carries an ionization chamber (I) .This arm also can be rotated about the same vertical axis as the crystal. The position of this arm can be determined by a second vernier (not shown in the figure).

The ionisation chamber is connected to an electrometer (E) to measure the ionization current. Hence we can measure the intensity of the diffracted beam of X- Rays, diffracted in the direction of the ionization chamber. S_3 is a lead slit, to limit the width of the diffracted beam. In practice, the crystal table is geared to the ionization chamber so that the chamber turns through 2θ when the crystal is turned through θ.

Working.

To begin with, the glancing angle θ for the incident beam is kept very small.

The ionisation chamber is adjusted to receive the reflected beam till the rates of deflection are maximum. The glancing angel (θ) and the intensity of the diffracted beam (I) are measured. The glancing angle is next increased in equal steps, by rotating the crystal table. The ionisation current is noted for different glancing angles. The graph of ionisation current against glancing angel is drawn. The graph obtained is as in Fig.2 and is called an X-ray Spectrum.

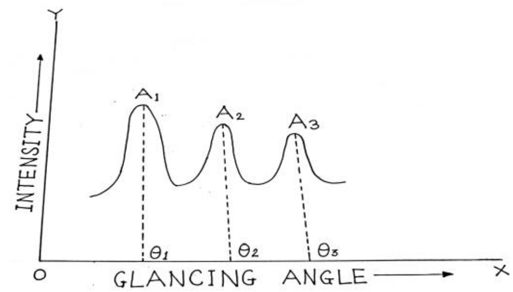


Fig. 2

The prominent peaks A_1, A_2, A_3 refer to X- rays of wavelength λ. The glancing angles θ_1,θ_2,θ_3 corresponding to the peaks A_1, A_2, A_3 are obtained from the graph. It is found that $\sin\theta_1 : \sin\theta_2 : \sin\theta_3 = 1: 2: 3$. This shows that A_1, A_2, A_3 refer to the first, second and third order reflections of the first, second and third order for another wavelength (λ_2). Thus Bragg experimentally verified the relation

$$2d \sin\theta = n\lambda.$$

The wavelength of X –rays is determined by using the equation $2d \sin\theta = n\lambda$. The glancing angle θ is experimentally determined as explained already for a known order. If d is known , λ can be calculated.

Example 1. *The spacing between principal planes of NaCl crystal is* 2.82 Å. It is found that first order Bragg reflection occurs at an angle of 10°. What is the wavelength of *X-rays* ?

Sol. By Bragg equation, $2d \sin\theta = n\lambda$.

Here, $d = 2.82 \times 10^{-10}$ m; n=1 and $\theta = 10°$. λ = ?

$$\boldsymbol{\lambda = \frac{2d\ sin\theta}{n} = \frac{2\ X\ (\ 2.82\ X\ 10^{-10})Sin\ 10°}{1} = 0.98\ X\ 10^{-10}\text{m}}.$$

Example 2. Bragg's spectrometer is set for the first order reflection to be received by the detector at glancing angel of 9° 18'. Calculate the angle through which the detector is rotated to receive the second order reflection from the same face of the crystal.

Sol. Let λ be the wavelength of X- rays . Let θ_1 and θ_2 be the glancing angles for the first and second orders. Then,

$$2d \sin \theta_1 = 1\lambda \text{ and } 2d \sin \theta_2 = 2\lambda.$$

$\therefore$ Sin $\theta_2 = 2 \sin \theta_1$, or sin $\theta_2 = 2 \sin 9°18' = 0.3232$ or $\theta_2 = 18° 48'$

Now, $\theta_2 - \theta_1 = 9° 30'$. Hence the crystal is rotated through an angel 9°30'. The angel through which the detector is rotated is twice the angle through which the crystal is rotated. Therefore , the angle through which the detector is rotated from first order to second order = $2 (\theta_2 - \theta_1) = 19°$
