# Quadrant II – Transcript and Related Materials

**Programme: Bachelor of Commerce(First Year)**

**Subject: Mathematics**

**Paper Code: CAC101**

**Paper Title: Commercial Arithmetic-I(CC4)**

**Unit: I**

**Module Name: Testing of validity of an Argument (1)**

**Module No: 18**

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**Notes:**

An argument is said to be *valid* if the conclusion is true when its all premises are true. An argument is valid if and only if its conclusion follows from its premises. Otherwise argument is said to be *invalid* or *fallacy.*

i.e., all the premises are true and the conclusion is false.

To check the validity of an argument we use truth tables. Also a result called **Fundamental Principle of Argumentation.**

**Critical Rows**

The rows of the truth-table are called *Critical Rows* for which all the premises are True.

In the critical row if the conclusion is true then the given argument is valid.

Otherwise argument is said to be invalid.

**Example 1.**

Find the critical rows in the truth-table of the following argument:

$$p \vee q\ ,\quad \sim p \quad \vdash \quad \sim q$$

**Sol.** Let $P_1 : p \vee q \;\; and \;\; P_2 : \;\sim p$ be the premises and Q : $\sim q$ , the conclusion.

Then the truth table will be of the type,

| | | $P_1$ | $P_2$ | Q | |
|---|---|---|---|---|---|
| $p$ | $q$ | $p \vee q$ | $\sim p$ | $\sim q$ | |
| T | T | T | F | F | |
| T | F | T | F | T | |
| F | T | T | T | F | √ critical row |
| F | F | F | T | T | |

The critical row exists in the third row of the above table as all the premises (Highlighted truth values) are true.

**Example 2.**

Construct the truth-table for the following argument and find the critical rows in the following argument;

$$\sim p \vee q\ ,\ \ q,\ \ p \rightarrow q \quad \vdash \ p$$

Here, $P_1 :\ \sim p \vee q$ , $P_2 : q$ *and* $P_3 : p \rightarrow q$ are the premises and the conclusion is Q : $p$

| | $P_2$ | | $P_1$ | $P_3$ | Q | |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim p \vee q$ | $p \rightarrow q$ | $p$ | |
| T | T | F | T | T | T | √ critical row |
| T | F | F | F | F | T | |
| F | T | T | T | T | F | √ critical row |
| F | F | T | T | T | F | |

In the above truth table, we find that the first and third rows as critical rows.

**Method to check the Validity of an Argument using Truth-table:**

**Step 1.** If the given argument is in verbal form, convert it into the symbolic form.

**Step 2.** Identify the premises and conclusion.

**Step 3.** Prepare the truth-table showing the truth values of all the statements.

(simple statements, premises, conclusion)

**Step 4.** Mark the critical rows where the premises are all true.

**Step 5.** If the conclusion is true for each critical row then the argument is valid.

**Step 6.** If the conclusion is false for the critical rows (i.e., false even in one critical row) then the argument is invalid.

**Example 1.**

Test the validity of the following argument:

$p \wedge q\,, \quad \sim p \rightarrow q \quad \vdash \quad q$

Here, premises are $P_1 : p \wedge q \;\; and \;\; P_2 : \;\sim p \rightarrow q$

and the conclusion is Q : $q$

| | | | $P_1$ | $P_2$ | Q | |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $p \wedge q$ | $\sim p \rightarrow q$ | $q$ | |
| T | T | F | T | T | T | √ critical row |
| T | F | F | F | T | F | |
| F | T | T | F | T | T | |
| F | F | T | F | F | F | |

In the above truth table all the premises are true in the first row.
Therefore first row is a critical row.
Since the conclusion is true in the first critical row,
∴ the given argument is valid.

**Example 2.**

Check the validity of the following argument:

$$p \to q\,, \quad \sim p \vee q \quad \vdash \quad \sim q$$

Here, premises are $P_1 : \; p \to q \;\; and \;\; P_2 : \;\; \sim p \vee q$

conclusion is Q : $\sim q$

We prepare the truth table and mark the critical rows.

| | | | $P_1$ | $P_2$ | Q | |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $p \to q$ | $\sim p \vee q$ | $\sim q$ | |
| T | T | F | T | T | F | √ critical row |
| T | F | F | F | F | T | |
| F | T | T | T | T | F | √ critical row |
| F | F | T | T | T | T | √ critical row |

We observe that in the above truth table there are three critical rows, first, third and fourth. But only in the fourth critical row conclusion is true.

∴ the given argument is invalid.

Finally, we can summarize that,

- the Truth-tables are used to test the validity of the arguments in mathematical logic.
- valid arguments preserve truth which we have noticed by testing the arguments.