Relationship between AM, HM and GM:

For any positive real numbers in the given series we always have the following relation between AM, HM and GM which is given by

AM ≥ GM ≥ HM

The equality condition holds true if all the observations in the series are identical.

Example: Find the AM, GM and HM for the following data and compare the three means.

  (i)   2, 8

Soln: AM $= \frac{2+8}{2} = 5,$ GM $= \sqrt{2 \times 8} = \sqrt{16} = 4$ HM $= \frac{2\times2\times8}{2+8} = \frac{32}{10} = 3.2$

  (ii)   (ii) 1, 4

Soln: AM $= \frac{1+4}{2} = 2.5,$ GM $= \sqrt{1 \times 4} = \sqrt{4} = 2$ HM $= \frac{2\times1\times4}{1+4} = \frac{8}{5} = 1.6$

  (iii)   (iii) 10, 30, 90

Soln: AM $= \frac{10+30+90}{3} = \frac{130}{3} = 43.33,$ GM $= \sqrt[3]{10 \times 30 \times 90} = \sqrt[3]{27000} = 30$ HM $= \frac{3}{\frac{1}{10}+\frac{1}{30}+\frac{1}{90}} = \frac{3}{13/90} = 20.77$

(iv) 4, 4

Soln: AM $= \frac{4+4}{2} = 4,$ GM $= \sqrt{4 \times 4} = \sqrt{16} = 4$ HM $= \frac{2\times4\times4}{4+4} = \frac{32}{8} = 4$