Quadrant II – Notes

Unit : Probability Theory

Module Name : Total probability and Bayes' theorem

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Notes

Total Probability

Let $\{E_1, E_2, \ldots, E_n\}$ be a partition of the sample space S, and suppose that each of the events $E_1, E_2, \ldots, E_n$ has nonzero probability of occurrence. Let A be any event associated with S, then

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + \ldots + P(E_n)\,P(A/E_n)$$

$$= \sum_{j=1}^{n} P(E_j)\,P(A/E_j)$$

The above equation states that event A is split into n parts, the P(A) is the sum of the probabilities of each part individually. This is called law of total probability.

Bayes' Theorem

In statistics and probability theory, the Baye's theorem is a mathematical formula used to determine the conditional probability of events.

If $E_1, E_2, \ldots, E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and A is any event of nonzero probability, then

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A/E_j)} \quad \text{for any } i = 1, 2, \ldots, n$$

Examples

A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Solution

Let B_1 be the event of choosing the first bag, B_2 the event of choosing the second bag and R be the event of drawing a red ball.

Then $\quad P(B_1) = P(B_2) = \frac{1}{2}$

Also $\quad P(R/B_1) = P(\text{drawing a red ball from I Bag}) = \frac{4}{8} = \frac{1}{2}$

and $\quad P(R/B_2) = P(\text{drawing a red ball from II Bag}) = \frac{2}{8} = \frac{1}{4}$

Now, the probability of drawing a ball from I Bag, being given that it is red, is $P(B_1/R)$

By using Bayes' theorem, we have

$$P(E_i/A) = \frac{P(E_i)\, P(A/\, E_i)}{\sum_{j=1}^{n} P(E_j)\, P(A/\, E_j)}$$

$$P(B_1/R) = \frac{P(B_1)\, P(R/\, B_1)}{P(B_1)P(R\,/\,B_1) + P(B_2)\, P(R/\, B_2)}$$

$$P(B_1/R) = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{8}}$$

$$= \frac{\frac{1}{4}}{\frac{3}{8}}$$

$$= \frac{1}{4} \div \frac{3}{8}$$

$$= \frac{1}{4} \times \frac{8}{3}$$

$$= \frac{2}{3}$$

$$= 0.67$$