

Quadrant II – Transcript and Related Materials

Programme: S. Y. B. Com

Subject: Mathematics

Paper Code: CAG102

Paper Title: Business Statistics-II (GE 5)

Unit IV: Interpolation and Extrapolation

Module Name: Interpolation and Extrapolation: Forward and Backward

Differences, Forward and Backward difference table.

Module No: 37

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Notes:

Interpolation and Extrapolation:

Definition:

➤ **Interpolation:**

Interpolation is defined as the technique of estimating value within the given range of the series.

➤ **Extrapolation:**

Extrapolation is defined as the technique of estimating value beyond the given range of the series.

Assumptions:

- There are no sudden jumps or falls in the values of the dependent variable for the period under consideration.
- The rise and fall in the dependent values should be uniform.

Finite differences:

Consider any function $y = f(x)$. Let us suppose that the equidistance values of the independent variable x are

$a, a + h, a + 2h, a + 3h, \dots, a + nh$.

Where a is the initial argument and h is the common interval.

Let the corresponding values of the independent variable y be,

$f(a), f(a + h), f(a + 2h), f(a + 3h), \dots, f(a + nh)$

Which are known as entries.

Forward differences:

The forward difference operator is Δ (Delta) and it is defined as

$$\Delta f(x) = f(x + h) - f(x); x = a, a + h, a + 2h, \dots$$

ie.

$$\Delta f(a) = f(a + h) - f(a)$$

$$\Delta f(a + h) = f(a + 2h) - f(a + h)$$

And so on. Where Δ is the first order forward difference.

The second order forward difference of $f(x)$ is defined by,

$$\Delta^2 f(a) = \Delta f(a + h) - \Delta f(a)$$

$$\Delta^2 f(a + h) = \Delta f(a + 2h) - \Delta f(a + h)$$

And so on. Similarly one can find third and higher order differences.

These differences can be expressed in the form of a table known as Forward difference table.

Forward difference table:

| X | f(x) | $\Delta f(x)$ | $\Delta^2 f(x)$ |
|----------|-------------|--|--|
| a | $f(a)$ | | |
| | | $\Delta f(a) = f(a + h) - f(a)$ | |
| $a + h$ | $f(a + h)$ | | $\Delta^2 f(a) = \Delta f(a + h) - \Delta f(a)$ |
| | | $\Delta f(a + h) = f(a + 2h) - f(a + h)$ | |
| $a + 2h$ | $f(a + 2h)$ | | $\Delta^2 f(a + h) = \Delta f(a + 2h) - \Delta f(a + h)$ |
| | | $\Delta f(a + 2h) = f(a + 3h) - f(a + 2h)$ | |
| $a + 3h$ | $f(a + 3h)$ | | |

Backward differences:

The backward difference operator is ∇ (Nabla) and it is defined as

$$\nabla f(x+h) = f(x+h) - f(x); x = a, a+h, a+2h, \dots$$

Backward difference starts with second entry $f(a+h)$. Here also subtract second entry with previous entry.

ie.

$$\nabla f(a+h) = f(a+h) - f(a)$$

$$\nabla f(a+2h) = f(a+2h) - f(a+h)$$

And so on. Where, ∇ is the first order backward difference.

The second order backward difference of $f(x)$ is defined by,

$$\nabla^2 f(a+2h) = \nabla f(a+2h) - \nabla f(a+h)$$

$$\nabla^2 f(a+3h) = \nabla f(a+3h) - \nabla f(a+2h)$$

And so on. Similarly one can find third and higher order differences.

These differences can be expressed in the form of a table known as backward difference table.

Backward difference table:

| X | f(x) | $\nabla f(x)$ | $\nabla^2 f(x)$ |
|------|---------|--------------------------------------|--|
| a | f(a) | | |
| | | $\nabla f(a+h) = f(a+h) - f(a)$ | |
| a+h | f(a+h) | | $\nabla^2 f(a+2h) = \nabla f(a+2h) - \nabla f(a+h)$ |
| | | $\nabla f(a+2h) = f(a+2h) - f(a+h)$ | |
| a+2h | f(a+2h) | | $\nabla^2 f(a+3h) = \nabla f(a+3h) - \nabla f(a+2h)$ |
| | | $\nabla f(a+3h) = f(a+3h) - f(a+2h)$ | |
| a+3h | f(a+3h) | | |

Example:

Construct the forward difference table for the following data.

| x | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|-----|--------|---------------------------------|---------------------------------|--------------------------------|
| 1 | 5 | | | |
| | | $\Delta f(a) = 8 - 5 = 3$ | | |
| 2 | 8 | | $\Delta^2 f(a) = 1 - 3 = -2$ | |
| | | $\Delta f(x + a) = 9 - 8 = 1$ | | $\Delta^3 f(a) = 1 - (-2) = 3$ |
| 3 | 9 | | $\Delta^2 f(x + a) = 2 - 1 = 1$ | |
| | | $\Delta f(f + 2a) = 11 - 9 = 2$ | | |
| 4 | 11 | | | |