Quadrant II – Glossary

Paper Code: CAC102

Module Name: Graphical Solutions of Linear Programming Problems with two variables only: Minimization of the objective function subject to the constraints

Glossary of terms/words:

Bounded feasible region:
Feasible region is bounded if it can be enclosed within a circle.

Constraints:
The restrictions or the limitations on the given decision variables are called as constraints. They can be linear inequalities or linear equations in terms of decision variables.

Corner point:
Any vertex of the feasible region is called corner point.

Decision variables:
Are the quantities that can be controlled. They are always non- negative. In any Linear Programming Problem, our aim is to find the values of these variables so that we get optimum(i.e. minimum/maximum) value of the objective function.

Feasible region:
A region common to all the constraints and which also satisfies the non-negative restrictions on the decision variables is called feasible region.

Linear programming:
Linear programming is a technique for determining an optimum schedule of interdependent activities in view of the available resources.

Objective function:
A linear function of decision variables that we wish to minimize or maximize is called objective function. It is expressed as $z=ax+by$ (where a and b are constants & x and y are decision variables).

Optimal Solution:
Any point in the feasible region which optimizes (i.e. minimizes or maximizes) the given objective function is called optimal solution.

Unbounded feasible region:
Feasible region is unbounded if it cannot be enclosed within a circle.

Possible misconceptions/clarification

Case Studies and Additional Examples/Illustrations

Solve graphically:
Minimize $z = 10x + 4y$
Subject to:
$4x + y \geq 120$
$3x + 3y \geq 180$
$x \geq 0$ and $y \geq 0$

Solution:

$z = 10x + 4y$ → Objective function
$4x + y \geq 120$
$3x + 3y \geq 180$ } Constraints
$x \geq 0$ and $y \geq 0$ → Non-negative restrictions
Where x and y are decision variables

Step 1: Represent each constraint on the graph.
Since in any Linear Programming problem , $x \geq 0$ and $y \geq 0$, graphical solution of the problem is always confined to the first quadrant. So we restrict to the first quadrant.
To represent the constraint $4x + y \geq 120$, we first consider the equation $4x + y = 120$.

By substituting x=0 in $4x + y = 120$, we get,
$4 \times 0 + y = 120$
$\therefore \quad 0 + y = 120$
$\therefore \quad y = 120$
Let us call this point as A(0,120).

By substituting y=0 in $4x + y = 120$, we get,
$4x + 0 = 120$
$\therefore \quad 4x = 120$
$\therefore \quad x = 30$
Let us call this point as B(30,0).

To represent the constraint $3x + 3y \geq 180$, we first consider the equation
$3x + 3y = 180$.

By substituting x=0 in $3x + 3y = 180$, we get,

$$3 \times 0 + 3y = 180$$
$$\therefore \quad 0 + 3y = 180$$
$$\therefore \quad 3y = 180$$
$$\therefore \quad y = 60$$

Let us call this point as C(0,60).

By substituting y=0 in $3x + 3y = 180$ we get,

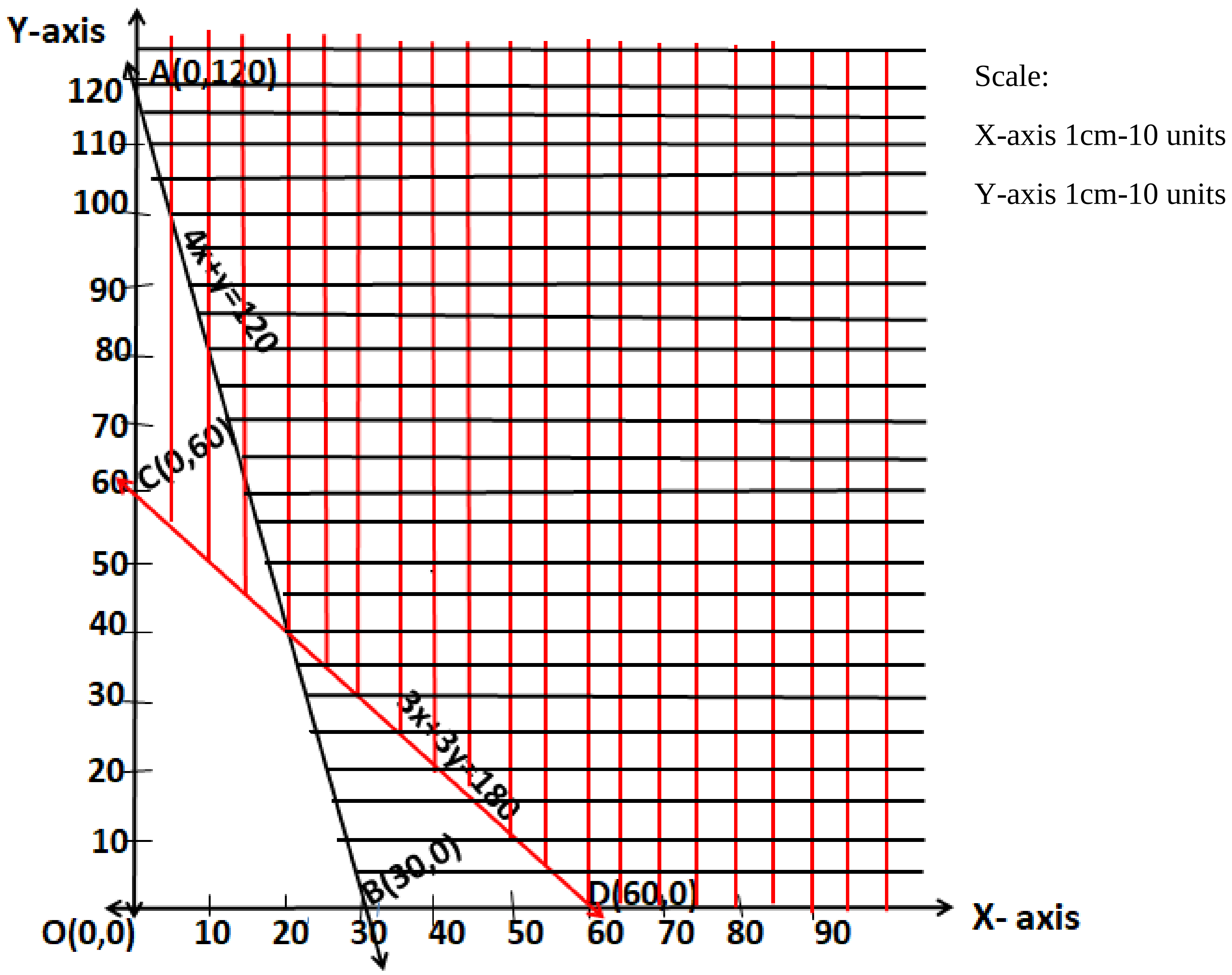
$$3x + 3 \times 0 = 180$$
$$\therefore \quad 3x + 0 = 180$$
$$\therefore \quad 3x = 180$$
$$\therefore \quad x = 60$$

Let us call this point as D(60,0).

After this, the points A(0, 120) , B(30, 0), C(0,60) and D(60,0) are plotted on a XY plane.

Next join the points A(0,120) and B(30, 0) to obtain the line 4x+y=120.
Also, join the points C(0, 60) and D(60, 0) to obtain the line 3x+3y=180.

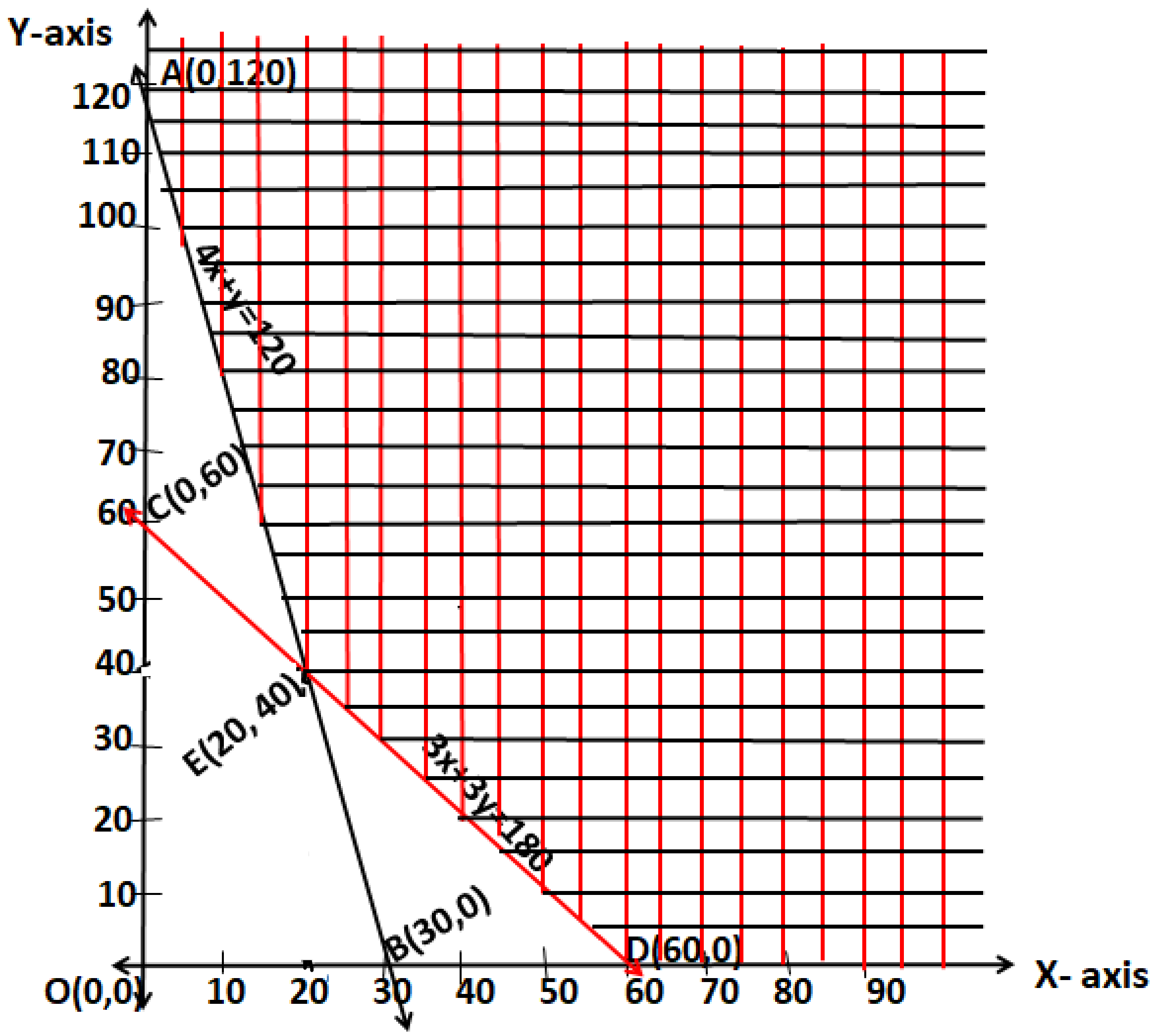
The line 4x+y=120 and the non-origin side of the line 4x+y=120 together represent the constraint $4x + y \geq 120$. So starting from the line $4x + y = 120$, we shade away from the origin (Refer graph no. 1).
The line $3x + 3y = 180$ and the non-origin side of the line 3x+3y=180 together represent the constraint $3x + 3y \geq 180$. So starting from the line 3x+3y=180, we shade away from origin (Refer graph no. 1).
For both the constraints we restrict the shading to the first quadrant because of non-negative restrictions on the decision variables.



Graph No. 1

Step 2: Identify the feasible region.

Infinite region to the right of AED (Refer graph no. 2) is the region common to all the constraints and it also satisfies the non-negative restrictions. So this region is feasible region. In this case feasible region is unbounded.



Graph No. 2

Step 3: Determine all the corner points of the feasible region.

Points A ,E and D are the corner points of the feasible region. Coordinates of A and D are already known. Please note that point E is the point of intersection of the lines 4x+y=120 and 3x+3y=180. The coordinates of point E can be obtained by mere inspection or by solving the two equations 4x+y=120 and 3x+3y=180 simultaneously by using elimination method or Cramer's rule. So after solving both the equations simultaneously we get the corner point E as (20, 40).

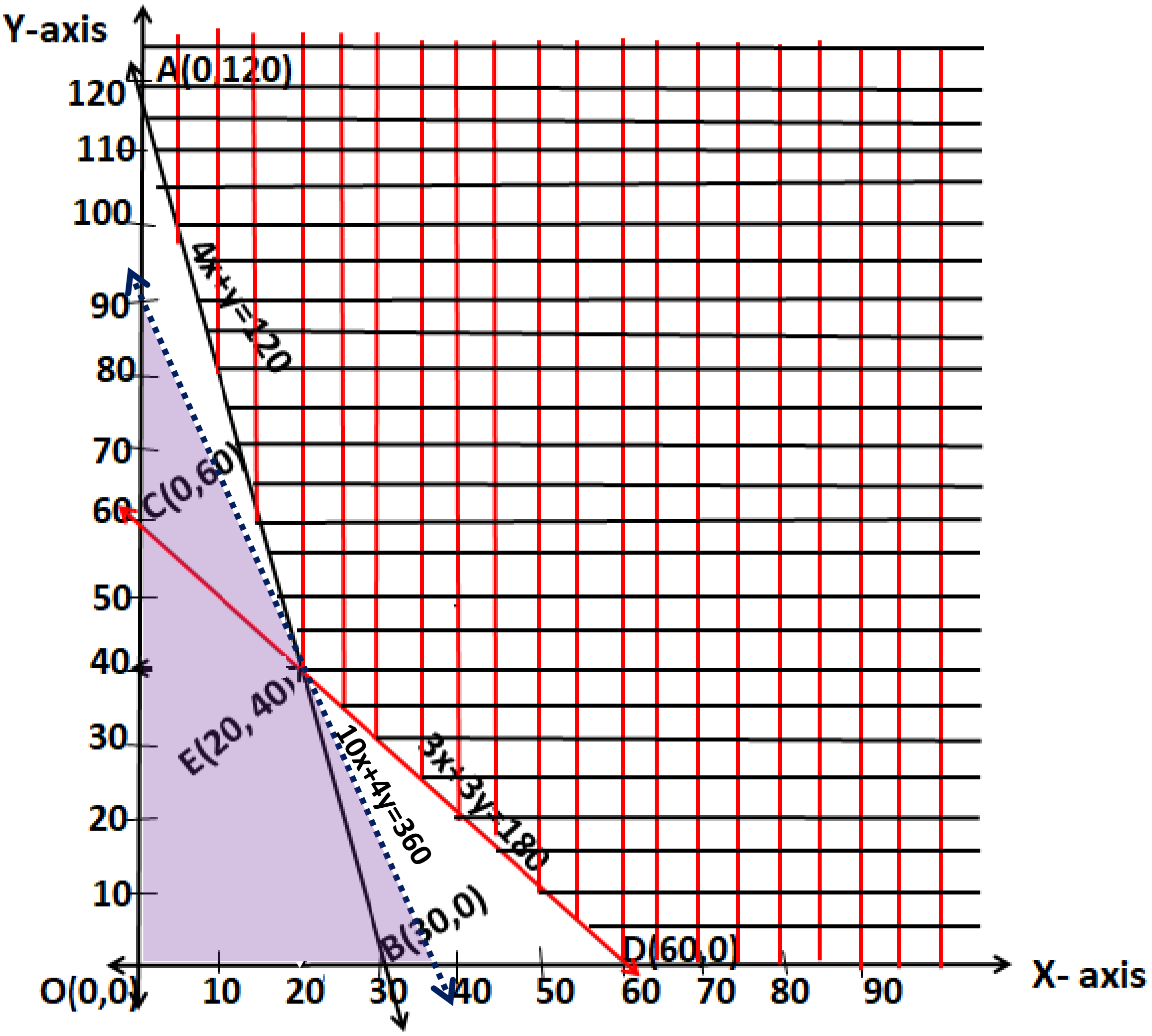
Step 4: Find the value of the objective function at each of these corner points.

Table No. 1

Corner points	$\mathbf{z = 10x + 4y}$
A(0,120)	$10 \times 0 + 4 \times 120 = 0 + 480 = 480$
E(20, 40)	$10 \times 20 + 4 \times 40 = 200 + 160 = 360$
D(60, 0)	$10 \times 60 + 4 \times 0 = 600 + 0 = 600$

Step 5:

From the above table it can be seen that 360 is the smallest value of z at the corner point E(20, 40). But since the feasible region is unbounded, 360 may or may not be minimum value of z. For that we need to check if there exists any point (x,y) in the feasible region at which objective function is having value lower than 360. For that we represent the inequality $10x+4y<360$ on the graph and check if any point satisfying $10x+4y<360$ lies in the feasible region. If such a point exists, then 360 will not be the minimum value of the objective function. In that case minimum value of z doesn't exist. Otherwise 360 will be minimum value of z. In graph no. 3, purple coloured shaded area represent the set of all the points in 1st quadrant which satisfy the strict inequality $10x+4y<360$.



Graph No. 3

From graph no. 3, we observe that the purple coloured shaded region doesn't intersect with the feasible region. This implies that, the region represented by $10x+4y<360$ doesn't have any point common with the feasible region. Therefore we can conclude that minimum value of z is 360 and that occurs at the point E(20, 40).