

Hello Student, in our first year B

Sc physics course PYC 101.

Mathematical method and mechanics.

And electrical circuit theory section one.

In this section one is

Mathematical Method and Mechanic.

In this section unit one Matrices

and determinants and linear equations.

The name of the 5th module

is a system of linear equations.

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Let us begin with the course outline so

over here we will be doing in this module.

Eigen values and properties of

Eigen values at the end of this

module you will be able to calculate

the eigenvalues of a matrix.

Now what is Eigen values?

Now?

Just remember eigenvalues are

the roots of the equation clear.

Whenever you should do find the roots

of this equation the similar way

it is a proper roots of the matrix

or we can say even say it is a

characteristic roots of our matrix.

So let  $A$  is Equal to  $A_i$ .

$J$  be square matrix,

then  $A X$  is equal to  $\lambda X$ ,

where  $\lambda$  is known as eigenvalue

of matrix  $A$ . And  $X$  is a column matrix.

This  $X$  is a column and

$\lambda$  is a eigenvalues.

Basically the other characteristic roots.

So now this equation is already familiar.

Only with you have this  $\lambda$ .

So this is matrix  $a$  that is  $a_{11}$ ,

$a_{12}$  till  $a_{m n}$  this is unknown which is a column

matrix  $X_1 X_2 \dots X_n$  multi is equal to  $\lambda$ .

Time this now what you do is that you

can find the equation from this matrix,

so they have to do you just multiply

this row first row with this column.

So you get this.

$1 \times 1$  plus  $a_{12} \times 2$  plus  $a_{1n}$  now

over here what is there you have

only one column so you all.

This will multiply 1 so

this column will multiply.

This row will multiply once with this column.

This will multiply with lambda.

This will multiply once so you get.

$a_{21} \times 1$  plus  $a_{22} \times 2$  plus  $a_{2n}$  xam,

and similarly the last one which is equal to.

This one multiplied.

This is a scalar multiplication.

OK, now this number multiplied by these

$X_1 \times X_2 \times X_3$  will become  $\lambda \times X_1$ ,

$\lambda \times X_2$ ,  $\lambda \times X_n$ .

Now whenever equal to sign is

there between two matrices.

The first row is equal to 1st

row of this matrix,

so we create them.

So this equation is equal to  $\lambda$ ,

Timex then this equation is equal under Timex

to this equation is going to number 10.

Exam so now we have to

do rearrange the terms.

Know like terms you bring together so

this a  $X^1$  and this  $X$  one is a like

term so bring them together so it will

become a  $11 - \lambda X$  one rest.

Everything will remain the same so that

is a  $12X^2 + \lambda XN$  equal to 0.

Similarly this will come at

this position that is  $X^2$  so a.

$22 - \lambda XN$  and so on.

So now this equation 3 can

be represented or simplify

by writing. This way that is more

of  $A - \lambda$  equal to zero.

Other than solving this here would

have to do to solve all this equation.

Find the values of  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  here.

Also have to do, but it is more.

Modam, sign and most employees order to.

Do you get a matrix minus  $\lambda$

Time  $I$  and  $I$  is your identity

matrix which you already familiar.

Since we're taking three cross three metrics.

So this is  $100010001$  and  $\lambda$  is

$\lambda$   $000$ ,  $\lambda$   $000$ ,  $\lambda$ .

When you subtract with this with the

A matrix, and since you see all

other terms you accept the element,

so when you subtract it will

carry minus sign.

$a_{11} - \lambda$ ,  $a_{13}$ ,  $a_{11} - \lambda$ .

$a_{21}$ ,  $a_{22} - \lambda$ ,  $a_{22} - \lambda$ ,

then  $a_{23}$ ,  $a_{31}$ ,  $a_{32}$ ,  $a_{33} - \lambda$

$\lambda$  is equal to 0.

So now we have to do.

Once you get this you just have

to solve like a determinant.

Rest it will get the answers

for these three Lambda.

Now one thing we should remember.

Suppose you are given a two

cross two matrix you will get.

Two Lambda value and that means

it has two characterized roots

if it has three cross.

Three matrix like this here you will get 3.

Characteristic roots Now this

equation over here it is called

characteristic equation of a matrix

and the roots of this equation

are called characteristic roots.

Some properties of these eigenvalues

any square matrix  $A$  and its transpose

$A^T$  has the same eigenvalues.

Next one the sum of eigenvalues of a

matrix is equal to trace of a matrix

and the third one the product of the

eigenvalues of a matrix is equal to.

Determinant of a matrix.

So let us solve some problem will

understand find the eigenvalues of a

matrix  $A$  which is equal to minus 5,

two  $2 \times 2$ .

Now what you have to do,

you just write the characteristic

equation that is  $A$  minus,

$\lambda$  times  $I$ .

Now since it is a  $2 \times 2$  cross to

matrix so  $I$  will be  $1001$  matrix

multiplied with the  $\lambda$   $00 \lambda$ .

So it will come.

This is a first so minus 5 minus  $\lambda$ .

So it will remain the same.

This would change all the diagonal

element will carry this minus  $\lambda$ .

In order to do your to solve

like a determinant.

So this full term multiply by

this full term minus this.

So minus 5 minus Lambda in 2 -- 2

minus Lambda minus 2 into 2 is 4,

which is equal to 0.

So it will have a simplified this

into this this in term' this

into this and this term this.

You will get this Lambda's square

plus seven Lambda. Plus 10 and minus 4.

Now if you simplify further,

you will get Lambda's square plus

seven Lambda plus 6 equal to 0.

How to solve this?

This is very simple which you have

already learned like that that

splitting the middle term correct.

Now splitting the Middle term

will split the middle tem it will

get back to this question answer.

Or you can use quadratic formula method.

So what you have to do for splitting

the middle term we just multiply this

number with this so you get 6 correct.

Now six. How will you get 6 ones

are six when you add them and get 7.

So you have one and six.

So  $\lambda$  plus one and  $\lambda$

plus 6 equal to 0.

Whenever you have this is equal to zero.

That means  $\lambda$  plus one equal

to 0 and  $\lambda$  plus 6 equal to 0.

So we will get  $\lambda$  is equal to minus.

One number is equal to minus six.

Therefore the eigenvalues of this

matrix is minus one and minus six.

Next one we will solve three across three matrix.

So this is  $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ .

So it turns it into three cross three matrices,

so we will get minus at the diagonal.

So  $2 - \lambda$ ,  $1 - \lambda$ ,  $1 - \lambda$ ,

$1 - \lambda$  equal to 0,

so determined to find out.

Take the first term.

They seem to this so that is this

second term minus this in two terms.

So you get this one minus  $\lambda$  plus

zero and the last term it will get one

$\lambda$  minus the status 0 minus 020.

Simplify, it will get one minus  $\lambda$ ,

$\lambda$ , minus three,

and  $\lambda$  minus one equal to zero.

When is simplified these you get 113,

so the eigenvalues are 113 OK.

See since it is coming both same,

you have to mention that they're

all saying that 113 OK.

Next one find the characteristic root of

a matrix 3216 same way so it will come.

Three 2 -- 6 so we will multiply

this into this minus this into this.

So we will get here.

$\lambda$  is the  $\lambda$  minus  $\lambda$ ,

so 3 minus  $\lambda$  into 6 minus.

$\lambda$  is equal to two which is 0.

Therefore,

$\lambda^2 - 9\lambda = 16$ .

And if you solve it using splitting

the middle term, you won't get,

so you have to use the quadratic

formula method,

which is  $-\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$ ,

where  $a = 1$ ,  $b = -9$ , and  $c = 16$ .

This will be in this year.

then substituted get their number is

equal to  $9 \pm \sqrt{23}$ .

These are the reference for this module.

OK, thank you.