Quadrant II - Transcript

Welcome students I'm Placida Pereir, Assistant professor in physics from Government College Of Arts, Science and Commerce, Quepem. So today's title of the unit is elementary vector algebra and the name of the module that we will be studying today is basis vectors and components magnitude of a vector and unit vector.

So the outline of this module we have basis vectors and components magnitude of a vector and unit vector.

At the end of this module, a student will be able to add and subtract vectors in their component form and also calculate the magnitude of a vector.

So let's begin. Let's start with basis vectors and components.

Consider three vectors let's say vector e1, e2 and vector e3 which do not lie in the same plane. If we have such three vectors which do not lie in the same plane, then it is possible to write another vector P in a 3-dimensional space as scalar multiples of them. That is, the vector P can be written as P1 times the vector e1.

Plus P2 times the vector e2 plus P3 times the vector e3. thus vectors e1, e2 and e3 are said to form a basis for the

three dimensional space. Also,scalars, P1P2, and P3 which can

be positive, negative or zero are called the components of the

vector P with respect to the basis. Hence we can say that

vector P has been resolved into components.

Also, we have to know that the basis vectors used have

to be mutually perpendicular and in general basis set must

have the following conditions satisfied. That is, there has

to be as many basis vectors as a number of dimensions.

Also, the basis vectors must be linearly independent.

A vector in three dimensional space requires three components

to describe its direction and magnitude letters in space.

The basis vector is denoted as i kept in the x

direction, the basis vector is denoted by j and in the y

Direction and the basis vector is denoted by k in z direction.

Also, the basis vectors i, j and k can be represented in the Cartesian

coordinate system by 1, 0,0. that is the i basis vector.

The j basis vector is denoted by 0,1,0

and the k basis vector along the Z

direction is denoted as 0,0,1.

Thus, now, since we've already established the basis vector for the Cartesian coordinate system, we can state any vector

P in terms of its components in three dimensional space. That is vector

the P is equal to px times basis vectorіI plus py times the basis vector j plus

Pz times the basis vector k. px is the component of the

vector P along the X direction. i is the basis vector along the

X direction. py is the component of the vector P along the Y

direction. j is the basis vectors along the Y direction, pz is the

component of vector P along the z direction and k is the basis vector along

the z direction . If we have any displacement in space

it may be thought of as the sum of the displacement along

the X, Y & Z directions.

Now let's consider a simple example. Let's say if I have a

vector a written in its component form that is, ax

times basis vector i plus ay times j plus az times k.

I, j and k vector are the basis vectors for the

Cartesian system. Also we have another vector B written in its

component form as bx extends i plus

by times j plus bz times k. To obtain the addition of the two vectors in their

component form, it can be shown as a vector in the sum of a and b.

I substitute the vector A in its component form plus the vector B

in its component form .The sum of vector A plus vector B in its component form is nothing but the sum components of vector A and vector B

Along the X direction times, the basis vector plus the components

of a of vector A and vector b in Y direction times the basis

vector along the y direction plus the component of vector A

plus the component of vector B along the z direction times

the basis vector along the z direction. That gives us a

ax plus bx times i plus ay plus

by times j plus az plus bz times k and

also the difference can be obtained by subtracting the two

vectors as how we had done in of previous case. You just

substitute the vector A in the component form subtracted by the

vector B in its components.

The subtraction is obtained by subtracting the X component of

vector A minus the X component of vector B times the basis

vector I plus the y component of vector A minus the Y component

vector B times the basis vector

J plus the z component of vector A minus the Z

component of vector B times the basis vector along the

Z direction.

Let's take an example.

Given two vectors, say vector c and d.Vector C is given by 2i plus 4j plus 3k.

And vector D is equal to

i plus 3j plus 2k. So what we have to do is we

have to find out the sum of vector C + 2 times the vector D

and also obtain the vector c minus vector D. So let's start

by finding out what is the sum of vector C.

plus twice the vector D, so I have substituted the vector C in

its component form as that is 2i plus 4j plus 3k

Plus two times the vector , the vector D is given as i

plus 3j plus 2k. So, to obtain the sum we

open the bracket,that is, I'll have 2i

plus 4j plus 3k..

I multiply two to the vector D I get 2i plus 6j plus

4k l and the sum is obtained is by adding the X

components to the X components we get 4i plus 10j plus 7k.

So this is the

sum of vector C + 2 times the vector D. Next let's go to the subtraction

We have to calculate what is the subtraction of vectors C

minus vector D. So I substitute the vector C as it is that it's

2i plus 4j plus 3K minus the vector D that is

I plus 3j plus 2k. The result is equal to

I plus j plus k.

Next, let's try to find out what is the magnitude of a vector. We already know that in order to specify a vector, we need magnitude and direction. So how do we find the magnitude of a vector? Let's first define what we understand by the magnitude with reference to vector quantity. The magnitude of a a vector is nothing but the measure of its length. This analogy is very useful for displacement vectors. Magnitude is better described for example by strength of vectors such as Force or by speed for velocity vectors. In general, the magnitude of a vector p is denoted by the modulus of P.

And in terms of a 3 dimensionalCartesian coordinates, the magnitude of P is given as

Modulus of p is equal to.

The root of square of x component of p plus squared of y component of P plus square of z component of the vector p. Let's take a simple example. Let's say if the velocity of an object is given by the vector 2i plus 3j plus 2k.

Then I want to obtain what is the magnitude of this velocity? We already know that the magnitude of velocity is nothing but Speed.

In order to find that out, I want to know what is the magnitude of velocity. So if

I want to know the magnitude,velocity and just take the

modulus of v.

Which is equal to square root of the square of the X

component of velocity plus squared of y component

velocity plus square of z component of velocity.

Substituting I get sqrt of 2 square plus 3 square plus 2 squared which gives

Square root of 17 and the unit is meter per second.

Next, let's go into the unit vector. What is a unit vector?

A vector whose magnitude equals unity is called in unit vector

In general the unit vector in direction P is usually

denoted as P cap.

And can be evaluated as this.

If i want to find out the unit vector

that is associated with any vector. It will be p cap is

equal to the vector P divided by the modulus of the vector p.

The unit vector is a useful concept becauses a vector, if

written, is the product of scalar Lambda into the unit vector.

Then we have the magnitude as Lambda and the direction is

P cap. Thus the magnitude and the direction are

explicitly separated.

Let's take an example. Find a unit vector that is parallel to the vector C which is equal to i plus 2j.

So the unit vector parallel to the vector C which is equal

I plus 2j is given by the formula for the unit

vector which will be equal to the vector C divided

by the magnitude of the vectorC. So let's find out what is the

magnitude of the vector C. So we know that the equation

to calculate the magnitude is given by modulus of vector c must equal

to the square root of squared of the X component of the vector C plus

the square of the Y component

of vector C which gives us sqrt 1 squared plus 2 squared which is

equal to square root of 5.

Now let's substitute this value in the equation for finding the

unit vector. So we have the unit vector C is equal to the vector

C divided by the modulus or the magnitude of vector C which is equal to

I plus 2j divided by the magnitude, which is equal to

Square root of five, giving

1 by square root of 5 times i plus 2 by square root 5 times j.

These are the references.

Thank you.