Quadrant II - Transcript

Welcome students I'm Placida Pereira, assistant professor in physics from Government, College of Arts, Science, and Commerce capping. So the title of the unit is elementary vector algebra and the name of the module that we will be covering today is dot and cross product of vectors and their physical interpretation.

So at the outline we have the multiplication of vectors in that we will be covering dot product and the cross product and in the end of this module the student will be able to carry out dot and cross product of vectors.

So to begin with, let's define what is a dot product or scalar product. If the product of two vectors result in a scalar, then the product is called a scalar or a dot product. The scalar product, which also is called the dot product of any two Vectors is denoted by

vector A dot vector B and is given by the formula the dot product of A dot B is equal to the magnitude of the vector a Magnitude of vector B times cos of theta where theta is less than equal to π and greater than equal to 0. Here Theta is the angle between the two vectors which are either placed tail to tail or head to head.

So we can observe it in this illustration . Let's

say if I have a vector A and

another vector B. The angle between the two vectors is given

by Theta.

Then the projection of vector B on vector A is given as B times

Cos theta. I can write the product, the dot product of

vector A dot B is equal to magnitude of vector A

magnitude of vector B times cos of the angle between

the two vectors A & B.

Also, from the definition of the scalar product, we can state

that A dot B is equal to 0. Is necessary and sufficient

condition for the vector A to be perpendicular to the vector B,

since if a is perpendicular to vector B, the angle will be 90

degree. If I take that angle and substitute in the formula of

the dot product cos of 90 is

equal to 0, giving me

the dot product equal to 0 or the other condition for a dot

To be zero is that either vector a is equal to 0 or the vector B is

equal to 0.

Since the Cartesian basis vectors, i, j and k are mutually orthogonal. They Satisfy the following equations.

That is, the dot product of i into I is equal to 1.

The dot product of j into j is equal to 1 and the dot

product of k into k is equal to 1.

Also, the dot product of i dot j which is equal to j dot k is

equal to k dot i is equal to 0.

Because the angle is 90.

Let's say if we have any two vectors in terms of its

components and the basis vectors which are mutually

perpendicular. Let's say I have a vector A r in component

form as ax basis vector. I plus ay

which is the component along the y-direction times j, which

is the basis vector along the Y-direction plus az which is the

component along the z

Direction. K is the basis vector along z direction. I have another vector B which is written bx

Times i kept plus by times j plus bz times k, then

the scalar product of the two vectors A and the vector B is

given by A dot B so I just substitute the vector A in its

component form that is ax times i plus ay times j plus

az times k.

Also , the component form of B is bx times i plus by times j kept plus bz times k. So what we'll get is A dot

B will be equal to ax times bx plus ay times by plus az times bz

because they've already established that i times i is equal

to 1 j dot j is equal to 1 and k dot k is equal

to.

As the name implies, the scalar product has a

magnitude but no direction.

The simplest example of scalar product is the work

done in this case, work done is nothing but the force into

the displacement vector r,so the work done in moving the

point of application of a constant force through a

displacement r.Also, we have to know that the displacement

if the displacement is perpendicular to the direction

of force, then no work is done because the angle would be 90

in that case.

The scalar product is commutative and distributive

over addition. That is, if I Have two vectors, vector a and

vector b , the dot product of vector a dot b is equal to

the dot product of vector B dot vector A. The

distributive property states that vector A dot the sum of

vector B plus vector C is equal to A dot B + A dot C.

Next, let's go into the cross product. The cross product is

also called as the vector

product. The vector product of two vectors, that is vector A and vector

B is denoted by A cross B.

The vector product is defined to be a vector of magnitude given

by magnitude of the vector A times magnitude of vector B times the

sine of the angle between them and the direction of the

resulting vector is perpendicular to the direction

of vector A and vector B. Also Here is a formula for the

magnitude of the vector a crossB which is equal to magnitude of

a magnitude of B times the sine of the angle between the two vectors.

Also we have to know that the direction of the

resultant vector, since it's a vector product, is found by

rotating the vector A into vector B through the smallest

possible angle.

Also, that the sense of rotation is

that of a right handed screw that moves forward in

the direction of A cross B..

So if I have a vector A and B and the angle

between them is theta, the resulting

vector A cross B is in the direction perpendicular to both the vector A and B.

So by this we can say that the vector A vector B and

the vector product of A cross B form a right handed set.

So the directions, how do we obtain the directions of the

vector product? The relative directions in a vector product

is provided by a right hand rule. The first two fingers and the thumb

are held as nearly mutually perpendicular as possible. If

the first finger is pointed in direction of the first vector

and the second finger in the direction

of the second vector, then the thumb gives the direction of

the vector product.

An important property of the vector product,

is the distributive property.That is, if I have three

vectors, let's see tire vector A vector B and vector C, then

the distributive property of the vector product states that the

sum of vector A plus vector B cross product of C

gives us a cross c plus b cross c.

Also, another important thing to note is that the

vector product is anticommutative. That is

b cross a is equal to the negative of a cross b.

The vector product has another useful property, that is, if a

cross B is equal to 0, then A is parallel or antiparallel to b

unless either of them is zero

Also a cross a equals 0.

An example of the use of the vector product is for finding

the area of a parallelogram given by two vectors, let's say with

sides A and B, then the area of the parallelogram is given as the

the magnitude of the cross product

of vector A and vector B.

Another example of the vector product if there is a solid body

rotating about an axis that passes through the origin with

an angular velocity given by the vector Omega, then we can

describe this rotation by vector Omega having a magnitude

given by magnitude, modulus of Omega and points along the axis

of rotation. The direction of Omega is the forward direction

of a right handed screw rotating in the same sense as the body.

The velocity of any point on the body with the position vector R

is given by vector V, which is equal to the cross product of

Omega into the position vector r.

Since the basis vectors i, jand k

are mutually perpendicular, unit vectors forming a right handed set.Their vector products can be

stated as I cross I is equal to J cross. J Cross j is

equal to K cross k which is equal to zero. Since the

angle is 90 degree.

Next also we have to know that I Cross J is equal to minus J

cross I will give us the unit vector k and

if I take the cross product of J cross k which is

the same as taking the cross product of minus K cross j

giving us the unit vector I.

Or the basis vector and the cross product of K cross I is

equal to minus I cross K giving us a vector J.

the product of two vectors, let's say vector A and

vector B is given in terms of the components in terms of the

basis set i,j and k is as follows. Let's Say if I have the cross product

A cross B, then I can state it is a ay which is the

component along the Y direction times bz said which is the component

of the vector b along the Z Direction minus az which is

the component of vector a

long the Z direction times by which is the component of the

vector B along the Y direction times the basis vector I plus

The the product of the az ,which is the component of the vector A along the Z direction times bx, which is the component of vector B along the X direction minus ax which is the component of the vector A along the X direction times bz which is the component of the vector B along the z direction times j, which is the basis vector along y direction plus ax which is the component of vector A along the X direction times by which is the component of vector B along the Y Direction minus ay, which is the component of the vector A along the Y direction times bx which is the component of vector b along the X direction times k which the basis vector along the z direction. In determinant form it can be written as follows equals be determinant of i, j and k and we substitute the components or vector A as ax , ay and az and the component of vector B as bx, by and bz after which will obtain the same equation as stated.

An example. Let's consider given two vectors, vector A, which is equal to 3i plus 2j minus K and another vector B given as 2i plus 3j plus 3k. Let's evaluate A dot B and A cross B.

So A dot B I have substituted the value of the vector A.

That is 3i dot 2i plus 2j dot 3j

giving me 6 + 6 -- 2 which is equal to 10.

Also, A cross B written in determinant form as i, j and k

Substitute the components of vector a as 3,2 and -1

And substitute the compliments of bector B as 2, 3 and 2

Giving the result i minus 8j plus 5k.

These are the references

Thank you.