

This is Unit 2 complex numbers. The name of this module is complex numbers and notations of complex numbers. This is the outline of this lecture. Will see general form of complex number, complex conjugate of a complex number, Basic operations. Involving complex numbers such as addition, subtraction, multiplication, and division. After this lecture you will be able to add, subtract, multiply and divide complex numbers and calculate the conjugate of a complex number. We denote a complex number by the letter  $Z$ . A Complex number is a number of the Form  $x + iy$  where  $x$  &  $y$  are real numbers. And the value of  $i$  is under root of minus one, so we can write  $z$  is equal to  $x + iy$ . So as you can see, a complex number has two parts. The first term is a real number and the second term is an imaginary number. We get complex number as a solutions of quadratic equation. Let us take an example of a quadratic equation. We have  $z^2 - 4z + 8 = 0$ . We can find the two roots of a quadratic equation using the formula that is  $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . If you substitute the values of  $a$ ,  $b$  &  $c$  in that formula will get. This. And if you do, the algebra will get.  $z$  is equal to  $2 \pm \sqrt{4 - 32}$ . Minus one by two. As we have noted, will write this under root of minus one as  $i$ , so we get  $z$  is equal to  $2 \pm 2i$ . The first term in this equation is a real term. It's a real number. The second term in this equation is an imaginary term, the full solution. Is the sum of real term an imaginary term and is called as a complex number. A complex number has a real part, and an imaginary part. But either of these parts can be 0. So if I have a complex number  $5 + 6i$ , I can say that 5 is a real part and six  $i$  is the imaginary part. If I have just seven as a complex number, I can say that the real part is 7. And the imaginary part is 0. If I have minus three  $i$  as a complex number, I can say that the real part is zero and the imaginary part is minus three  $i$ .

Next we have complex conjugate of a complex number. If we have a complex number,  $Z$  is equal to  $X + iY$  the complex conjugate of that number.  $Z$  is a number  $X - iY$ . And it is denoted by  $\bar{z}$  or  $Z^*$ . So to get complex conjugate of a complex number, we replace the sign of all the  $i$ 's from positive to negative. And vice versa, let us take an example, find the complex conjugate of a complex number.  $Z$  is equal to  $W^2 + 2X$ , where  $W$  is equal to  $X + 3i$  so will substitute the value of  $W$  here, so we get  $Z$  is equal to  $(X + 3i)^2 + 2X$ .

$6y + 2x$ . Now to find the complex conjugate we replace each  $i$  by minus  $i$ . So we get the complex conjugate.

Of the number  $Z$  as that star is equal to  $x - 3i$ , I will do the power six  $y - 2x$  addition and subtraction of complex numbers. We add and subtract complex numbers by adding their real and imaginary parts separately. So if I have  $z_1$  and  $z_2$  as two complex numbers in these, all the  $x$ 's are the real parts and  $y$ 's are the imaginary part. So if I want to add  $z_1$  plus  $z_2$ , I will add the real part that is  $x_1$  plus  $x_2$  and I'll add the imaginary part  $y_1$  and  $y_2$ .

Similarly we can do the subtraction  $z_1$  minus  $z_2$ .

multiplication of two complex numbers. We multiply complex numbers by expanding the brackets and noting that the value of  $i$  squared is equal to minus one. So let  $z_1$  is equal to  $x_1 + iy_1$  and  $z_2$  is equal to  $x_2 + iy_2$ . So these are the two complex numbers  $z_1$  into  $z_2$  is given by  $x_1 + iy_1$  into  $x_2 + iy_2$ .

I have to multiply each of the terms in  $z_1$  by each of the terms with  $z_2$  so I get  $x_1$  into  $x_2$ . Now there is no  $i$  in this, so this is a real part then I've  $x_1$  into  $iy_2$  and  $y_1$  into  $y_2$  and we note that  $i$  into  $i$  is  $i^2$  which has a value minus one. So I get minus here. Then the imaginary terms which are  $ix_2$  and  $x_1i$  and  $y_2$ . I'm writing this  $i$  common out here. As you can see here, the product is in the form  $X + iY$ , which is also a complex number. Next we have division of two complex numbers to divide complex numbers. We note that  $Z$  into  $Z^*$  that is a complex number into its complex conjugate is a real number, so we need to solve it to get rid of all the  $i$ 's in the denominator. To do this, we multiply the numerator and the denominator by the complex conjugate of the denominator. This is similar to the rationalization process. Let  $X + iy$  &  $U + iv$  are two different complex numbers. Now we have to divide  $X + iy$  by  $U + iv$ . Now I'm taking complex conjugate of the denominator that is  $U - iv$  and multiplying it by the numerator and denominator. In this case the denominator we get as a real number because this is a complex number into its complex conjugate. So we get  $u^2 + v^2$  in the numerator. We have to multiply by expanding the bracket. So we get  $Xu + yv + iYU - Xv - iYU$  And again, the product here. The final answer is also in the form of  $X + iY$ , which is a complex number. Let us take an example express  $Z$  is equal to  $3 - 2i$  /  $-1 + 4i$  in the form  $X + iY$ . So basically we need to

do the division here. So to do that I'm taking the complex conjugate of the denominator. It is  $1 - 4i$  and multiplying. The numerator and the denominator by the complex conjugate of the denominator. If I expand the bracket, I get all these terms and the denominator is nothing But  $z$  into that star. So we get minus one square Plus minus  $4^2$ . So finally, if we do the algebra, we get  $\frac{-11 - 10i}{17}$ . And again, final answer we get as  $\frac{-11 - 10i}{17}$  so this is again a complex number. These are the references. Thank you.