

This is Unit 2 complex numbers. The name of this module is complex planes and physical meaning of complex quantities. This is the outline of this module. Will see complex plane representation of complex numbers in argand diagram. Modulus and argument of complex number, then physical meaning of complex quantities. After this lecture you will be able to read complex numbers on an argand diagram, and calculate modulus and argument of complex numbers. In case of real numbers, we represent all the real numbers on a number line, so that line is an infinite 1 dimensional line. In case of real numbers, the real numbers are either positive, negative or zero. But in case of complex numbers which has the form,  $z$  is equal to  $x + iy$ . It has two parts that is real part and an imaginary part. So we cannot represent a complex number on a normal number line, so in case of complex numbers we need to represent under root of minus one. Now this under root of minus one is not zero. It is not a negative number neither it is a positive number. So instead of an infinite number line. We use an infinite number plane to represent complex numbers. So this plane is called as an argand plane. In the Argand plane, the horizontal axis that is  $x$  axis is called as the real axis. The vertical axis is called as an imaginary axis. We represent complex numbers in an argand diagram. Like we plot coordinates in a graph. Suppose if we have a complex number  $2 + 3i$ . I'll see where is 2 on the real axis and where is 3 on the imaginary axis and represent this complex number here. This is like we're plotting coordinates in a graph note that a complex number is a single number and not 2 numbers added together. Then we have modulus of complex number. The modulus of the complex number is the distance of corresponding point  $z$  from the origin in the Argand diagram. So basically, modulus of the complex number is the length of this line. If we know the distances. This  $y$  and  $x$  we can find the modulus of. Any complex number  $z$  as  $\text{mod } z$  is equal to  $\sqrt{x^2 + y^2}$ . argument of the complex number. The argument of complex number. We write it as  $\text{arg } z$ . Argument of  $z$  is the angle that the line joining the origin to the  $z$ . On the argand diagram makes with the positive  $x$  axis the positive  $x$  axis, that is the real axis. This anti clockwise direction is taken to be positive by convention. If we know the values of  $y$  and  $x$ . We can find argument that is equal to  $\tan^{-1} \frac{y}{x}$ . Let us take an example, find modulus and argument of the complex number.  $z$  is equal to  $2 + 3i$ . In this complex number, the value of  $x$  is 2 and the value of  $y$  is 3. So in the solution. The modulus is given by  $\text{mod } z$  is equal to  $\sqrt{x^2 + y^2}$

$\sqrt{2}$ . So we get  $\text{mod } z$  equals to  $\sqrt{2^2 + 3^2}$ , that is  $\sqrt{13}$ . Then the argument of the complex number  $z$  is given by  $\tan^{-1} \frac{Y}{X}$ . So we have  $Y$  is equal to three and  $X$  is equal to two. So we have the argument that is equal to  $\tan^{-1} \frac{3}{2}$ . physical meaning of complex quantities. Now in case of complex numbers. We make use of the fact that these numbers are 2D dimensional. That is, this numbers have real as well as imaginary dimension. Now using complex numbers, we can represent rotating physical system whose real value is also changing. We have one more type of representation of complex number that is polar form. So in case of polar form we use polar coordinates that is  $R$  and  $\theta$  instead of  $X$  &  $Y$  to represent complex numbers. Now this is polar representation of complex number. This  $r$  is the modulus of the complex numbers and this  $\theta$  is the argument of the complex number. I can use trigonometry and write. This distance  $Y$  as  $r \sin \theta$  and the distance  $X$  is  $r \cos \theta$ . So we have the general form of complex number, that is that is equal to  $X + jy$ . I'm substituting the values of  $X$  &  $Y$  here, so we get  $z$  is equal to  $R \cos \theta + j R \sin \theta$ . If I take  $r$  common out of the bracket we get  $z$  is equal to  $R (\cos \theta + j \sin \theta)$ . Next we get  $z$  is equal to  $R e^{j\theta}$ . this form is called as the polar form of complex number.  $e^{j\theta} = \cos \theta + j \sin \theta$ . This we get using the Euler's formula. Now in electric circuits. The voltage and currents. Changes sinusoidally with time, so we can represent them as complex numbers and represent both amplitude and phase of the variation as a single complex number. Again, in AC electrical circuits. Which contains ohmic, resistor, capacitor and inductance. We have complex resistances known as impedances. Which can be expressed as a complex numbers. So in case of the resistance  $R$ . We can write the impedance as  $Z_R$  is equal to  $R e^{j0}$ . So in this case the  $\theta$  value is zero. That means the phase is 0. In case of the capacitor the impedance  $Z_C$  is equal to  $\frac{1}{j\omega C} = \frac{1}{\omega C} e^{-j\frac{\pi}{2}}$ . So in this case. The phase is minus  $\frac{\pi}{2}$ . So this is nothing but a complex number  $R e^{j\theta}$  and similarly in case of the inductor  $Z_L$  is equal to  $j\omega L = \omega L e^{j\frac{\pi}{2}}$ . So here the argument  $\theta$  is  $\frac{\pi}{2}$  and the modulus is  $\omega L$ . These are the references. Thank you.