This is Unit 2 complex numbers. The name of this module is De moivre’s
theorem and roots of unity. This is the outline of
this module. Will see De moivre’s theorem and roots of
unity. After this lecture you should be able to calculate.
The NTH power of a complex number using De moivre’s theorem.
And calculate the NTH root of unity, the De moivre’s theorem
states that for any complex number z we can find the NTH
power of z, that is z power N is given by r power N
into cos n theta plus I sine N theta where N is integer. Will prove this.
We have z is equal to R E to the power I theta this is the
polar form of complex number. If I take Nth power on both the sides
we get z power N is equal to R E to the power ITheta whole power
N. If I take this n inside. We get r power N into
e to the power I n theta. If I expand this e to the power i n theta
using Euler's formula we get z power N is equal to r power N
into cos n theta into i sine n theta which is the statement
of De moivre’s theorem. So to take the NTH power of any
complex number, we take the NTH power of the modulus R.
And multiplied the argument theta by N. Let us take an example.
Find 1 by 2 + 1 by two I whole power 10, so this is the complex
number in the Form x plus iy. And we have to find the NTH
power of this complex number. So to find the NTH power of complex
number and to use the De moivre’s theorem we need to 1st
express this complex number in the form z is equal to R E to
the power I theta that is the polar form of complex number to do that we
need the values of R the modulus and theta that is the argument of
z. So to find the modulus R we have. Modulus z is equal to
under root X ^2 + y ^2. So here the value of X is half and value
of Y is also half. So we get the modulus mod z is equal to
under root one by two square plus one by two square which is
nothing but 1 by under root 2. Then to find the argument theta. Which is given by tan inverse of
Y by X. So will substitute the values of Y and X. So we get the argument of z
as tan inverse of half by half that is an inverse of 1
which has the value π by 4. So now we have written the
complex number in the form of polar form. That is r e to the power
I theta. so we have one by under root 2 E to the power I pi by 4.
We can also expand this e to the power i Pi by 4 using the Euler’s
formula. We get one by under 2 into cos pi by 4 plus I sine π by
four. Now if you want to find the 10th power of the complex
number here, the value of n is 10, so we use. The De moivre’s theorem.
So we'll take the N power of the Modulus that is 1 by under Root
2 whole to the power 10. And will multiply the argument theta by
10. So we write cos 10 pi by 4 plus I sine 10 by 4. If we

solve this, we get one by two to the Power 5 into cost 5 pi by 2 plus I sine 5 pi by 2. Now the cos 5 Pi by 2 has value zero, and Sine pi by two has the value one. So finally we get. Half plus half I whole to the power 10 is equal to 1 by 32 I next we have NTH root of a number an NTH root of any number z is a number R. Which, when raised to the power n yields z. So if This r is any root of N root of a number, if we. Take r power n we will get the number z or we can say that the NTH root of a number Z is equal to R. Finding the NTH root of unity. So in case of a complex number Z, if we have z square is equal to 1. So in this case the degree is 2. So this will have two solutions. If I take square root on both the sides I get z is equal to plus or minus one. Now to find the NTH root of unity we need to find the general solution of z power N is equal to 1 or unity. So we have z power N is equal to 1. Let's say this is equation one. This equation has any solutions because the degree is N. Now we have the Euler's formula that is E to the power i theta is equal to Cos Theta plus I sine Theta. I'm substituting the value of theta as 2π K, so we get e to the power Ik 2π is equal to cos K 2π plus I sine K2pi. Now the value of cos K 2π Where K is any integer. The value of Cos K 2π is 1 an sine. K 2π is 0. So we have e to the power IK. 2π is equal to 1. Let's say this is equation 2. Now from equation one and equation two, we get z power N is equal to e to the power iIK 2π is equal to 1. Now if I take square root of this equation, I get z is equal to. OK, if I'm taking the NTH root on both the sides I get z is equal to e to the power I K2 pi by N. hence the solution of the equation z power N is equal to 1 we have. Z is equal to 1. This is K is equal to 0 if we have K is equal to 1 we have the solution e to the power I 2π by N. And for K is equal to N -- 1. We have e to the power IN minus one 2π by N. So if we continued for the larger values of K, we do not get any new solutions since all the roots already listed are simply cyclically repeated for the higher n values, that is, for K is equal to n k is equal to equal to N + 1, K is equal to N + 2, etc. Let's take an example. Find the solution of the equation. Z power three is equal to 1. Now we have the power of z. That is, N is equal to three. That means we will get three solutions, so we have the formula z is equal to E to the power IK 2π by N. So will substitute the values of K Now, K is an integer for K is equal to zero. We have the first solution z One is equal to e to the power 0 into 2π. I by three, so this is e to the power zero that is 1 so z one is the first solution. That is 1. For k is equal to 1 we have z2 is equal to e to the power I into 1

into 2π by three so that is E to the power 2π I by three.
And for K is equal to two. We have z three that is E to the power four Pi. I by three if we try to find the next solution that is z four forK is equal to three. We get z 4 is equal to e to the Power I3 into 2π by three, which is equal to e to the power I 2 pi.
Which has the value one, so this is same as z one that means the solutions are repeating again, so these are the references. Thank you.