Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (First year)

Subject: Physics

Paper Code: PYC -101

Paper Title: Mathematical Methods & Mechanics and Electrical Circuit Theory

Module Name: Series and parallel resonance, Q factor and Bandwidth, Graphic representation of resonance

Module No: 26

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Notes:

- **Series resonance**

Given figure shows A.C. applied to a circuit containing inductance, capacitance and resistance in series.

The expression for current is given as

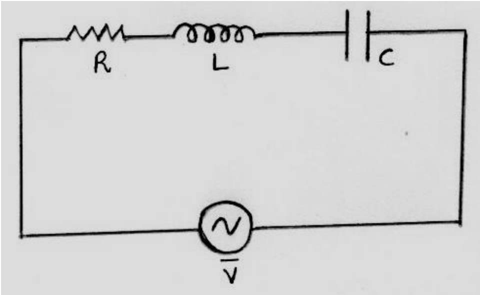
$$I = \frac{E_o}{Z}\sin(wt - \theta)$$

Therefore peak value for current is given as

$$I_o = \frac{E_o}{\sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}}$$

If $X_L = X_c$ that is $wL = \frac{1}{wC}$, $\theta = 0$ (I and E in phase)

In this case reactance is zero hence circuit acts like a purely resistive circuit.



In general impedance $\bar{Z}$ is always greater then R in series LCR circuit and hence the peak value of current I_{omax} is always less than $\frac{E_o}{R}$

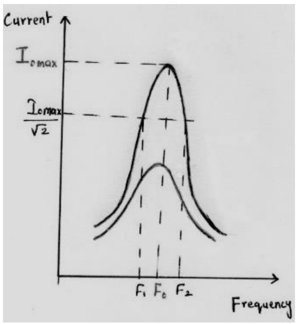
When Z = R which is minimum, I_o becomes maximum. This is called as **series resonance**.

At resonance $w_o L = \frac{1}{w_o C}$

$$\therefore w_o = \sqrt{\frac{1}{LC}}$$

$$\therefore F_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

At F_o current becomes maximum as shown in the given figure.



Hence we obtain resonance current I_{omax}

The frequencies corresponding to $\frac{I_{omax}}{\sqrt{2}}$ are called as cut-off frequencies.

The difference between the two cut-off frequencies F_1 and F_2 is called as **bandwidth**

$\Delta F = F_1 - F_2$

The significance of bandwidth is that the circuit responds effectively to the frequencies within the band. Thus series LCR circuit is exhibiting selectivity therefore it is called as acceptor circuit.

The sharpness of resonance depends upon the bandwidth.

Smaller the bandwidth sharper is the resonance, hence faster is the fall of current on both the sides of resonance frequency.

- **Q factor**

Voltage magnification also called as Q factor is defined as the ratio of the magnitude of the potential difference across the inductance to the applied EMF at resonance.

$Q = \frac{jwL \times I}{R \times I}$

$\therefore Q = \frac{w_o L}{R}$

- **Parallel resonance**

Consider an alternating EMF applied across an inductance L in parallel with a capacitance C as shown in the figure.

Current I = $\frac{E}{Z}$

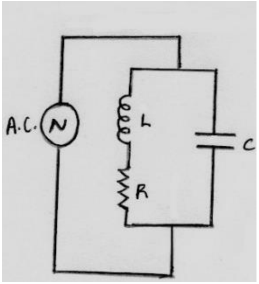
Where Z is total impedance

$\therefore \frac{1}{Z} = \frac{1}{R+jwL} + jwC$

$\therefore Y = \frac{1}{Z} = \frac{R+j(wCR^2+w^3L^2C-wL)}{R^2+w^2L^2}$

Where reciprocal of impedance is called as admittance Y

$$|Y| = \frac{\sqrt{R^2 + (wCR^2 + w^3L^2C - wL)^2}}{R^2 + w^2L^2}$$



When admittance is minimum, the impedance will be maximum and the current in the circuit will be minimum.

$$wCR^2 + w^3L^2C - wL = 0$$

$$\therefore w = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

therefore resonance frequency is given as

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

At the resonant frequency the current is minimum. Hence this type of circuit is called as rejector circuit.

Given figure shows the graph of circuit current against the frequency of applied EMF.