Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (Second Year)

Subject: Physics

Paper Code: PYC 103

Paper Title: Waves and Oscillation

Unit: 02

Module Name: Wave Equation and Solutions

Module No: 09

Name of the Presenter: Virroy V. Dias

Notes

Wave Motion

Wave motion is a form of disturbance which travels through the medium due to repeated periodic motion of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next.

When a stone is dropped into a pond containing water, waves are produced at the point where the stone strikes the water in the pond. The wave travels outward, the particle of water vibrates only up and down about their mean positions. Water particles do not move/travel along the wave. Similarly, when a tunning fork is set into vibration, it produces waves in air. The wave travels from one particle to the next but the particles of air vibrate about their mean positions.

It is essential to understand the concept of wave motion in the study of various branches in physics. Wave motion in general, refers to transfer of energy from one point to another point of the medium. Transference of various forms of energy like sound, heat, light, X—rays, radio waves, etc. takes place in the from of wave motion. For transference of energy through medium, the medium must possess the properties of elasticity, inertia and negligible frictional resistance. In all the waves, the particles of the medium vibrate about their mean positions. Hence, in the case of wave motion, it is not matter that is propagated but only state of motion, it is form of a dynamic condition.

Dynamic condition is related to momentum and energy.

To conclude, in wave motion momentum and energy are transferred or propagated.

Types of Waves

Transverse | Longitudinal

(a) Transverse wave

(b) Longitudinal wave

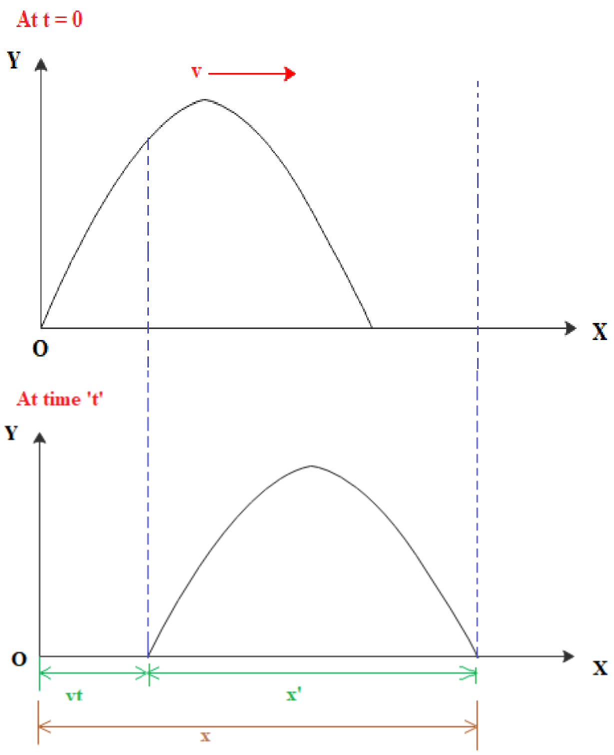
Particle of medium vibrate at right angles to the direction of propagation

Ex: Light waves

The Particles of medium vibrate along direction of propagation of the wave.

Ex: Sound wave

Equation of Simple Harmonic Wave



Consider a transverse pulse travelling from left to right along a stretched string with a velocity v.

At time 't=0'

The string is represented by

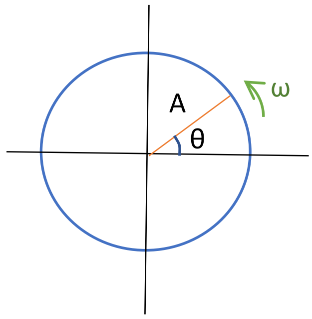
$$y = f(x) \quad ----(1)$$

At time ‘t’

$$y = f(x - vt) \quad ------(2)$$

This equation represents a pulse advancing from left to right.

Simple harmonic motion is given by



$$y = A \sin \theta$$

But,

$$\theta = \omega t$$

$$\Rightarrow \theta = \frac{2\pi}{T} t$$

$$\Rightarrow \theta = \frac{2\pi}{\lambda/v} t$$

$$\Rightarrow \theta = \frac{2\pi}{\lambda} vt$$

$$\Rightarrow \theta = \frac{2\pi}{\lambda} x$$

Therefore, we can write equation (1), as

At time t=0,

$$y = A \sin \frac{2\pi}{\lambda} x$$

and At time ‘t’, equation (2) can be written as

$$y = A \sin \frac{2\pi}{\lambda}(x - vt) \quad --------(3)$$

This is the equation of the wave propagating from left to right at time ‘t’.

Similarly, equation of a wave propagating from right to left can be written as

$$y = A\ sin\ \frac{2\pi}{\lambda}(x + vt) \quad --------(4)$$

We can also write eq. (3) and (4) as follows

$$y = A\ sin\ \frac{2\pi}{\lambda}(vt - x) \quad --------(5)$$

$$y = A\ sin\ \frac{2\pi}{\lambda}(vt + x) \quad --------(6)$$

Differential Equation of Wave Motion:

The general equation of a simple harmonic wave is

$$y = A\ sin\ \frac{2\pi}{\lambda}(x - vt) \quad -----(3)$$

Differentiating above equation w.r.t. 't'

$$\frac{dy}{dt} = A\left(\frac{-2\pi}{\lambda}v\right) cos \frac{2\pi}{\lambda}(x - vt) \quad -----(7)$$

Differentiating above equation w.r.t. 't'

$$\frac{d^2y}{dt^2} = -A\frac{4\pi^2}{\lambda^2}v^2\ sin\ \frac{2\pi}{\lambda}(x - vt) \quad ------(8)$$

To find the value of compression, differentiate eq. (3) w.r.t. '*x*'

$$\frac{dy}{dx} = A\frac{2\pi}{\lambda} cos\ \frac{2\pi}{\lambda}(x - vt)$$

Differentiate above eq. w.r.t. 'x'

$$\frac{d^2y}{dt^2} = -A\frac{4\pi^2}{\lambda^2}v^2\ sin\ \frac{2\pi}{\lambda}(x - vt) \quad --------(9)$$

Dividing above equations *(1) & (2), and rearranging, we* get

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

This represents the Differential equation of wave equation.

The general differential equation of wave motion can be wtitten as

$$\frac{\partial^2 y}{\partial t^2} = K \frac{\partial^2 y}{\partial x^2}$$

Where: $v = \sqrt{K}$

Thus knowing the value of K, the value of wave velocity can be calculated.

Problem 1) :

When a simple harmonic wave is propagated through a medium, the displacement of a particle (in cm) at any instant of time is given by

$$y = 10 sin \frac{2\pi}{100}(3600t - 20)$$

Calculate the amplitude of the vibrating particle, wave velocity, wavelength, frequency and time period.

Solution:

The general equation of simple harmonic wave is

$$y = A\ sin\ \frac{2\pi}{\lambda}(vt - x)$$

Comparing the given equation in the problem with general equation of harmonic wave, then

A=10 cm, λ=100cm, $v = 36{,}000 cm/s$ → wave velocity

$$\nu = \frac{v}{\lambda} = \frac{36000}{100} = 360\ Hz$$ → Time period

$$T = \frac{1}{\nu} = \frac{1}{360} = 2.77 \times 10^{-3}\ s$$

Problem 2) :

A wave is represented by the equation $y = 0.20\ sin\ 0.40\pi(x - 60t)$ where distances are in cm and time in seconds. Find (i) the amplitude, (ii) wave length, (iii) speed, (iv) frequency of the wave, (v) what is the displacement at x=5.5 cm and t=0.02s?

Solution:

The general equation of simple harmonic wave is

$$y = A\ sin\ \frac{2\pi}{\lambda}(vt - x)$$

Comparing the given equation in the problem with general equation of harmonic wave, then we get

(i) $A = 0.20\ cm$

(ii) $\frac{2\pi}{\lambda} = 0.40\pi$

$$\Rightarrow\ \lambda = \frac{2}{0.40} = 5\ cm$$

(iii) $v = 60\ cm/s$

(iv) $\nu = \frac{v}{\lambda} = \frac{60}{5} = 12\ Hz$

(v) $y = 0.20\ sin\ 0.40\pi(5.5 - 60 \times 0.020)$

$$\Rightarrow\ y = 0.20\ sin\ (1.72\pi) = 0.20 \times (-0.77) = -0.15\ cm$$
