Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (Second Year)

Subject: Physics

Paper Code: PYC 103

Paper Title: Waves and Oscillation

Unit: 02

Module Name: Superposition of Two Simple Harmonic Motions - III

Module No: 12

Name of the Presenter: Virroy V. Dias

Notes

Lissajous Figures

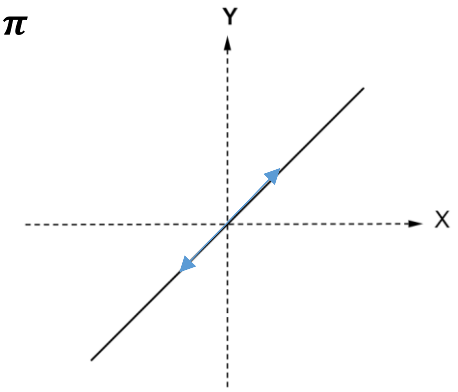
When a particle is influenced simultaneously by two SHM at right angles to each other, the resultant motion of the particle traces a curve. The curves are called as Lissajous figures.

The shape of the curve depends on the time period, phase difference and the amplitude of the two constituent vibrations.

Lissajous figures are helpful in determining the ratio of the time periods of two vibrations and to compare the frequencies of two tuning forks.

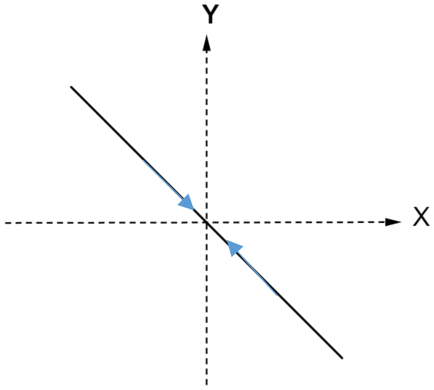
The Lissajous figures for **(b)** Perpendicular direction, same frequency (module #11) for different cases are as follows:

(i) If $\delta = 0 \ or \ 2\pi$



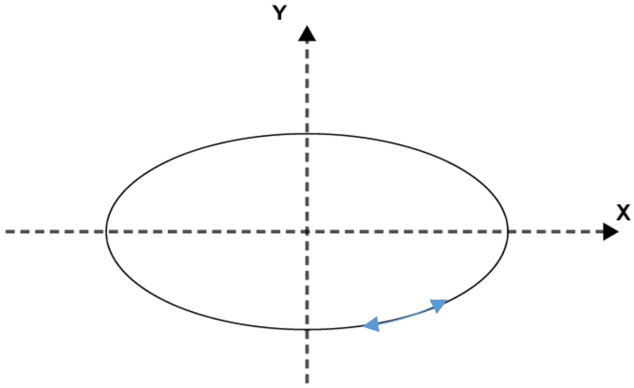
δ=0 or 2π

(ii) **If $\boldsymbol{\delta = \pi}$**



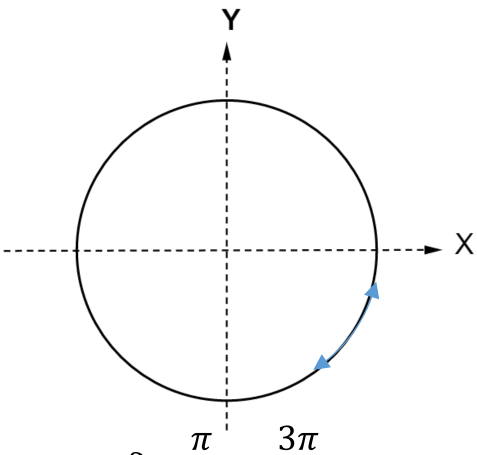
$\delta = \pi$

(iii) **If $\boldsymbol{\delta = \frac{\pi}{2}\ or\ \frac{3\pi}{2}}$**



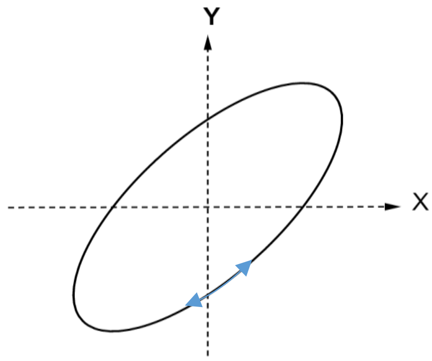
$\delta = \frac{\pi}{2}$

(iv) ***If $\boldsymbol{\delta = \frac{\pi}{2}\ or\ \frac{3\pi}{2}}$ and a=b***



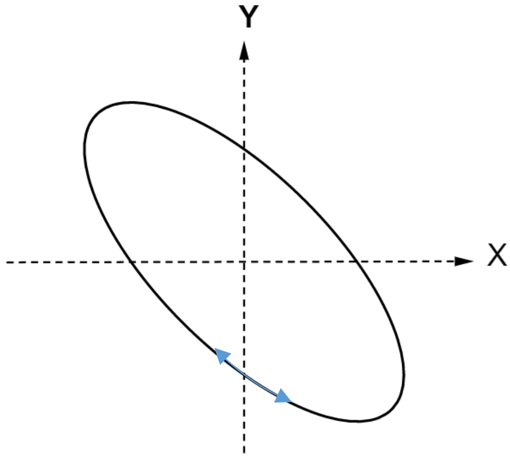
$\delta = \frac{\pi}{2}\ or\ \frac{3\pi}{2}$

(v) *If* $\boldsymbol{\delta = \frac{\pi}{4}}$ ***or*** $\boldsymbol{\frac{7\pi}{4}}$



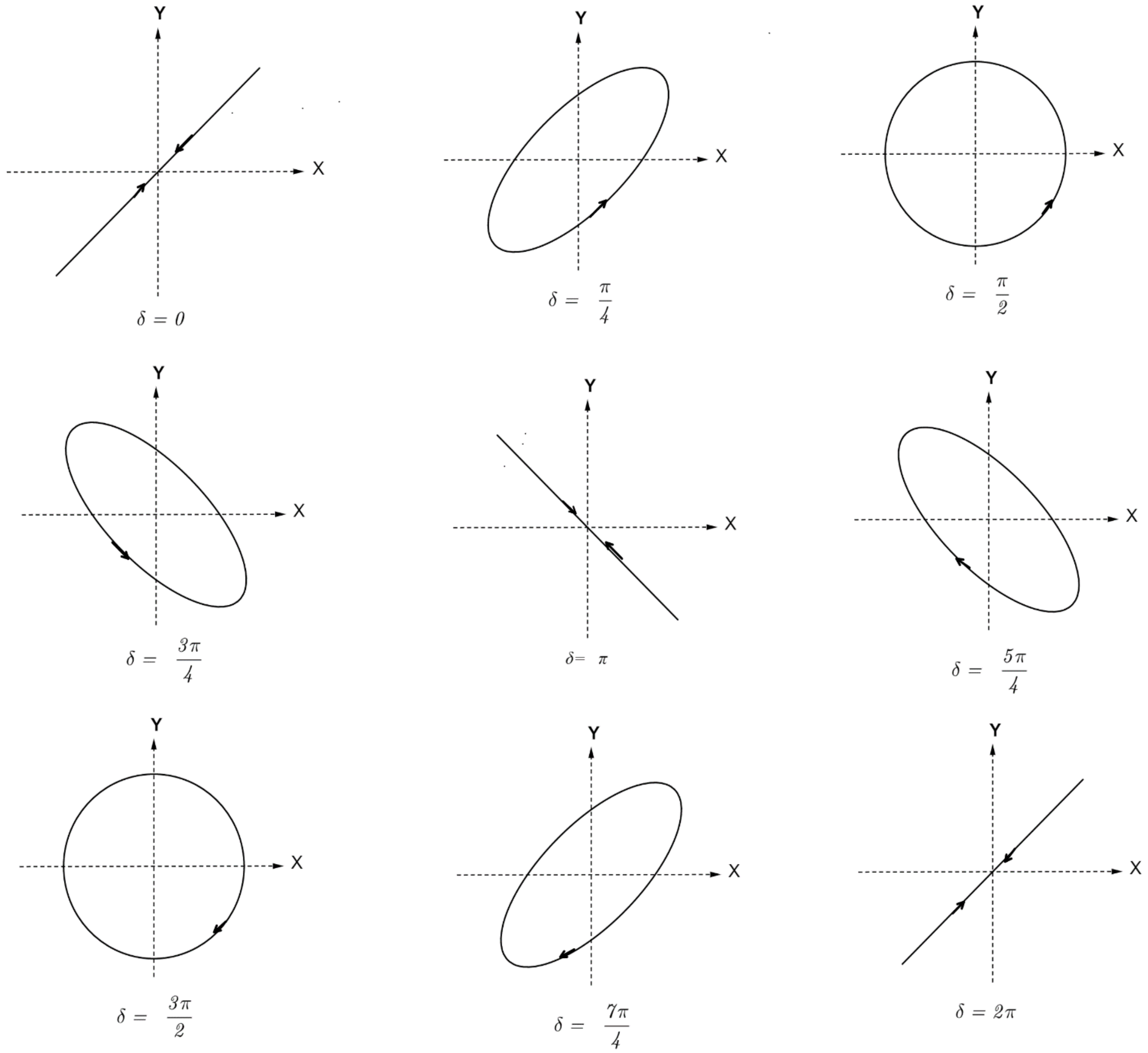
$$\delta = \frac{\pi}{4} \; or \; \frac{7\pi}{4}$$

(v) ***If*** $\boldsymbol{\delta = \frac{3\pi}{4}}$ ***or*** $\boldsymbol{\frac{5\pi}{4}}$



$$\delta = \frac{3\pi}{4} \; or \; \frac{5\pi}{4}$$

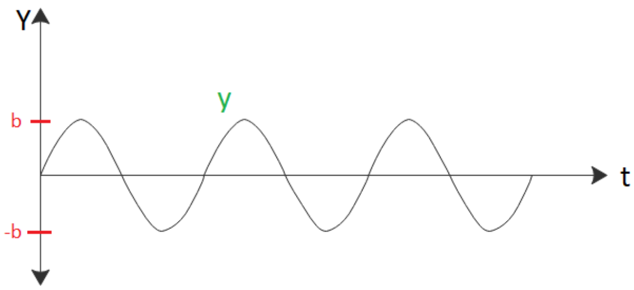
The direction of rotation (clockwise or anti-clockwise) of the particle may be obtained from the X and Y motions of the particle at time 't' is gradually increased.

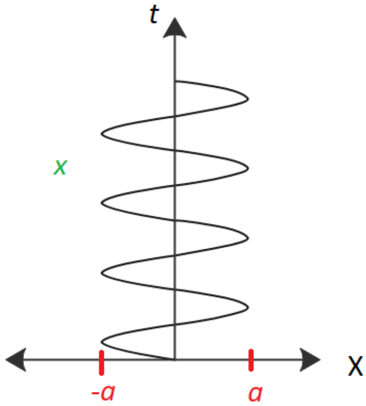


Lissajous figures for different phases difference of two simple harmonic motions ***which are perpendicular and have same frequency*****.**

(c) Perpendicular Directions, Different Frequency

Let $y = b\sin(\omega t) \quad ----(1)$ and $x = a\sin(2\omega t + \delta) ----(2)$





where: a and b are the amplitudes for motion along X and Y axis respectively. The phase difference between the two vibrations is δ .

In this case we consider the frequency ratio to be **2:1**.

Rearranging eq.(1)

$$\frac{y}{b} = \sin\omega t ----(3)$$

$$\Rightarrow \cos\omega t = \sqrt{1 - \frac{y^2}{b^2}} ----(4)$$

We can write eq.(2) as

$$\frac{x}{a} = \sin(2\omega t + \delta)$$

$$\frac{x}{a} = \sin 2\omega t \cos\delta \; + \; \cos 2\omega t \sin\delta$$

$$\begin{pmatrix} \sin 2a = 2\sin a \cos a \\ \cos 2a \; = (1 - \sin^2 a) \end{pmatrix}$$

$$\frac{x}{a} = (2\sin\omega t\cos\omega t)\cos\delta + (1 - \sin^2\,\omega\mathrm{t}\,)\sin\delta \quad ----(5)$$

Substituting values from (3) and (4) in above equation

$$\frac{x}{a} = \left(2.\frac{y}{b}.\sqrt{1 - \frac{y^2}{b^2}}\right)\cos\delta + \left(1 - 2\frac{y^2}{b^2}\right)\sin\delta$$

$$\left[\frac{x}{a} - \left(1 - 2\frac{y^2}{b^2}\right) sin\ \delta\right] = \frac{2y}{b} cos\ \delta \sqrt{1 - \frac{y^2}{b^2}}$$

$$\left[\left(\frac{x}{a} - sin\ \delta\right) - \frac{2y^2}{b^2} sin\ \delta\right] = \frac{2y\ cos\ \delta}{b} \sqrt{1 - \frac{y^2}{b^2}}$$

Squaring both sides, we get

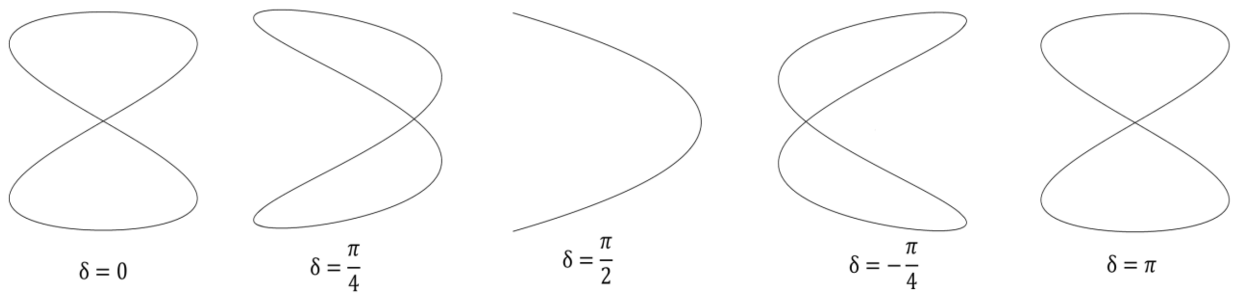
$$\left(\frac{x}{a} - sin\ \delta\right)^2 + \frac{4y^4}{b^4} sin^2\ \delta + 2\left(\frac{x}{a} - sin\ \delta\right)\frac{2y^2}{b^2} sin\ \delta = \frac{4y^2\ cos^2\delta}{b^2}\left(1 - \frac{y^2}{b^2}\right)$$

$$\Rightarrow \left(\frac{x}{a} - sin\ \delta\right)^2 + \frac{4y^4}{b^4}(sin^2\ \delta + cos^2\delta) - \frac{4y^2}{b^2}(sin^2\ \delta + cos^2\delta) + \frac{4y^2}{b^2}\frac{x}{a} sin\ \delta = 0$$

$$\Rightarrow \left(\frac{x}{a} - sin\ \delta\right)^2 + \frac{4y^4}{b^4} - \frac{4y^2}{b^2} + \frac{4y^2}{b^2}\frac{x}{a} sin\ \delta = 0$$

$$\boxed{\Rightarrow \left(\frac{x}{a} - \sin\delta\right)^2 + \frac{4y}{b^2}\left(\frac{y^2}{b^2} + \frac{x}{a}\sin\delta - 1\right) = 0} \quad ---(6)$$

Equation (6) represents the general equation of a curve having two loops. The resultant motion of the particle for different values of δ is given in figure below:



The Lissajous figures for two periodic motions of frequency ratio 1:2 for different values of δ.
