Hello Students, a warm welcome to you. Today's module is for the program Bachelor of Science, Second Year Subject Physics Semester 3 paper code. PYC 103 Section 1 paper title Waves and Oscillation.
The title of the unit is superposition of waves, module name, superposition of two simple harmonic motions, Part 3 module numbers is 12. I'm Virroy Dias assistant Professor Carmel College of Art, Science and Commerce for Women Nuvem Goa. The outline of this module is Lissajous figures. And superposition of two mutually perpendicular simple harmonic vibrations of different frequencies.
Learning outcome of this module is upon completion of this module, the student will be able to understand Lissajous figures, construct a resultant motion from 2 mutually perpendicular simple harmonic motions having different frequencies.
Now the first topic is Lissajous figures. Now what is this Lissajous figures?
When a particle is influenced simultaneously by two simple harmonic motions at right angles to each other. The resulting motion of the particle traces a curve. This curve is what are called as Lissajous figures. The shape of this curve depends on the time period, phase difference and the amplitude of the two constituent vibrations. Lissajous figures are helpful in determining the ratio of time periods of two vibrations and to compare the frequencies of two tuning Forks. Now this is the application, one of the application of Lissajous figures.
Now let us plot the Lissajous figures for two simple harmonic motions which are perpendicular and having the same frequency and acting on the particle. Now, we've already discussed this in our previous model where we had derived some equations for the general equation of an ellipse. So we found that the curves of the general equation for different cases to be as follows.
In the first case, if you remember, we had found the phase difference to be equal to 0 or 2π and we got a curve to be Y is equal B by A into X. now, This equation represents the equation of a straight line as depicted in the figure on your right. So if the particle is acted upon by two perpendicular simple harmonic wave motions having same frequencies, then the particle traces a path which is a straight line. That is, it moves along a straight line.
If the phase difference is equal to π then we get the curve to be equal to Y is equal to minus B by A into X, now this represents the equation of a straight line with a negative slope and the particle in this case moves along the line as shown in the figure. If the phase difference is equal to π by two or three Pi by two and the amplitudes are both the original harmonic motions are equal, then we get the equation to be X ^2 + y ^2 is equal to A square. This represents the equation of a circle. And the particle in this case follows a path in either clockwise or anticlockwise depending on the phase, that is the phase difference.
The next case we consider the phase to be equal to π by 4, seven π by 4, and we get this Equation. If we re-collect this equation is nothing but the equation of an oblique ellipse. As shown in the figure on the right. And the particle again will trace a path either in the clockwise or anti clockwise direction depending on the phase. That is the phase difference.
Similarly, in the last case, we had found. The phase difference would be equal to three π by four or five π by four, and again in this case also, we had found the resultant vibration to be an ellipse, an oblique ellipse. But the orientation over here is different compared to case number 5, and again in this case also the particle will either follow a clockwise or anti clockwise path depending on the phase difference.

These are the Lissajous figures for different phase differences of two simple harmonic motions which are perpendicular and have same frequency. So here I've denoted each phase separately. As you can see in the figure.

Now let us come to the last and final case of superposition of two simple harmonic motions.

In this case, we consider two simple harmonic motions which are perpendicular to each other and at different frequencies.

Let us consider the first motion would be Y is equal to B sine Omega T as can be seen in the figure below. Let's call this equation number one and the next harmonic motion to be X is equal to A sine two Omega T plus Delta as depicted in the figure below. Let's call these equations one and two.

Now in this equations A and B is the amplitude for the motion along X and Y. The phase difference between the two vibrations is Delta. Now if you notice here, there's a phase angle just for X. The phase difference within both these motions will be Delta.

In this case, if you notice, we have considered the frequency ratio to be 2 is to one. Now, just as in the previous case, this case also we want to discuss if we superposition two waves which are perpendicular and of different frequencies on a particle then what is the resultant motion of the particle. So that's what we want to find out.

So we can write the equation one as. Y by B is equal sine Omega T. Let's call this equation number 3. And using the trigonometric identity sine squared Omega T plus cos squared Omega T is equal to 1. I can write equation #3 as follows. Let's call this equation four.

We can write equation 2 as X by A is equal to this. So here again, I use trigonometric identities. Let's call this equation #5.

Now let us substitute the values of equation three and four in equation #5, where we have found here. equation three and four in the equation #5. And let us after substituting, square it and then rearrange it. So once we do that, we get this equation.

Let's call this equation, #6. Now this equation is nothing but the general equation of a curve having two loops. Now since, this equation is also made up of two perpendicular motions, we can plot the Lissajous figures for this as well. The Lissajous figures for two periodic motions of frequency one is to two for different values of phase difference Delta. Can be; is as shown in the figure below. Now if you notice here, there are two loops for this graph for some phases. That's what we got from this equation, equation #5. If you go back. This one we got from this equation #6. These are the references that I've used for this module. With this we come to the end of this module. An end of this unit. Thank you.