Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (Second Year)

Subject: Physics

Paper Code: PYS101

Paper Title: Network Analysis

Unit: 3 Transient Response of RL and RC circuits and RLC circuit to DC and AC signals

Module Name: Transient response of RLC circuit.

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Transcript

Welcome students.
I'll be taking the program on Bachelor of Science Second Year
Subject Physics. Semester 3.
Paper code. PYS101.
Paper title. Network analysis.
The title of the unit. Unit 3 response of RL RC and RLC circuits to DC and AC.
Module name Transient response of RLC circuit.
Module No. 16
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Outline is Transient analysis of RLC circuit.
Differential equations for RLC equation circuit .
Solution of differential equations for RLC circuit.

The learning outcomes. The student
Will understand the application of transient analysis to RLC circuit.
Be able to set up differential equations for RLC circuit.
Be able to understand on how to find complete solution to 2nd order differential equations for RLC circuit.
Student will be able to understand the meaning of the terms overdamped, underdamped and critically damped.

Transient response of RLC circuit as defined earlier.

Transient is a phenomena occurring during the transition from one stable state to another whenever circuit elements like inductor and capacitor or both inductor and capacitor are present in a circuit. In an RLC circuit we have inductor and a capacitor present.

So V is applied to the RLC circuit and we take the case during the charging.
So when the switch is closed at, say t is equal to 0.
Assuming that the inductor current at 0 plus is equal to inductor current at 0 minus and that is equal to 0 and the voltage across the capacitor at 0 plus is equal to 0 minus.
This continuity equation V(0) is equal to 0 and Q at time t equal to 0.
The capacity is fully discharged before the switch is closed.
The switch is closed at time t is equal to 0. Applying Kirchoff's voltage law to the complete circuit. V is equal to R I(t), where I(t) is the current flowing through the circuit. V_L is the voltage developed induce across the inductor L. And Vc the voltage across the capacitor C.
And that is given by V is equal to R into I plus L dI by dt plus Vc(t).
In terms, writing the equation in terms of single dependent variable. Yeah, and I put Qo equal to VC.
And we put I(t) is dQ by dt derivative of dQ with respect to dt, an dI(t) derivative of current with respect to time is equal to d square Q(t) divided by dt square.

So we putting these values in the equation for V we have V is equal to L d(square) Q(t) by dt (squared) plus R dQ(t) divided by dt plus Q(t) by C.
This is a second order differential equation.

Writing this equation equation in the standard form.
d square Q(t) by dt square plus 2b … this is a standard form of a second order differential equation.
And writing 2b is equal to R by L equal to 2b, K square is equal to 1 by LC, f is equal 2b by L.
So comparing these two equations, we have this.
The solution to this equation Q(t) is equal to e raise to minus bt. A e mt + B e m –t plus bracket plus Qo where we have m is equal to plus square root of the b square minus k square . and f by k square equals VC equal to Qo.
At t is equal to 0, Qo is 0, and substituting the values in the Q for Q(t)

we get A + B + Qo equal to 0 and which gives us A +B is equal to minus Qo OK.
Taking the derivative of this equation Q(t) and substituting for t equal to 0 and Q is equal to ….. and dQ(t) by dt at t(0) equal to zero,
we get a second condition. A -- B is equal to minus Qo b/m

And the first 2 A equal to minus Qo.(One minus b/m)
2B is minus Qo one minus b/m. OK.
An the general equation for Q(t) is given by this equation.
OK here we have m is equal to square root of b square plus k square.
So depending on the value of m.
We have three different conditions.
1 b squared greater than k square.
2. is b square is less than k square
3. is b square is equal to k square.

So taking the first condition, b square is greater than k square.
This is the case of overdamped circuit, here since this b square is greater than k square m is real, substituting for the values of b ^2 -- k squared, we get …..
We talk in terms of R^2 / 4 L^2 is greater then one by LC. So substituting these values for m in this equation we are we We show that Q(t) grows in an exponential fashion, asymptotically to Qo. Charging is now known as the deadbeat.

The second case, b square is less than k square, is an an example of underdamped circuit. OK, m in this case is imaginary and we write m in terms of value in terms of complex number minus….., square root of minus one that is j which gives us square root of k square minus b square = omega, and therefore m = j omega. The solution for this for this condition is given by this relation

Now as e raise j omega t is cos omega t plus sin omega t
& e raise minus j omega t is equal to cos omega t minus sin omega t,
substituting the values of A, B, e j omega t , e minus j omega t & m and rearranging the terms we come to the equation.
Q(t) is equal to e raise to minus bt Qo in bracket cos omega t + j. b/m sin omega t plus Qo
This can be rewritten as this equation
we right omega cos omega t plus b sin omega t
is equal to k in bracket sin alpha cos omega t plus cos alpha sin omega t.

OK, yes we have from here k is equal to…….
omega is equal to k sin alpha,
b is equal to k cos alpha
and omega is equal to b tan alpha.
The solution for this equation… this equation can be read written as
Q(t) equal to Qo one minus e raise to minus R upon 2L t k/Omega sin omega t plus alpha charge on the capacitor.

This equation simply indicates
that the charge on the capacitor oscillates about the
final value Qo with damped simple harmonic motion with amplitude of charge charge decays as per the term or as per the term e raise to minus.
R divided by 2 L in bracket t.
And the corresponding logarithm decrementt being RT divided by 2L.
So the period of the damped oscillations T is equal to 2 pie by omega.
The current I(t) which is equal to derivative or dQ(t)/dt is equal to

Qo k square e rise to minus R divided by 2L into t divided by omega
which is also oscillatory and having the same log decrement

The third condition when b square is equal to k square
Or b is equal to k. Is the critically damped circuit in that case m is equal to 0
substituting the value for m in the equation we have
Q(t) equal to Qo minus Qo e raise to minus bt one plus bt.
The capacitor charges to the final value of Qo in the shortest possible time.
And the current I(t) is given by this relation Qo b squared t e raise minus bt.
…stops flowing when the charge reaches its maximum value.

This is the nature of the change in the charge through the capacitor.
Charging current Q is equal to is given for underdamped it is oscillatory, for overdamped it quickly reaches is steady value Qo and for b is equal to k the critically damped. It's steady state value is reached in shortest time.

Transient response of RLC circuit.
This is capacitor discharge case.
We go to the same process and in this case we have assumed that the capacitor has been fully charged to Qo at t is equal to 0.
When the switch is closed at t is equal to 0. The voltage source is removed and writing Kirchoffs voltage equation for this circuit.

we have zero is equal to RI(t) plus V_L(t) plus VC(t), which is given by this.. substituting the value for V_L and VC.

In terms of single dependent variable we can write this equation ..differential equation as as written here ……
It is equal to the d square Q(t) to dt is the 2nd order differential equation plus…..
The solution is given by Q(t) equal to e raise to minus bt A e raise to mt plus B e raise to –mt,
this is a standard format for this equation,
substituting for the values of 2b and k square we have 2b is equal to R by L, k square equal to 1 by LC & f is equal to V by L,
so this is the solution for this equation where m in this case is plus b square – k square, f/ k square equal to VC is equal to Qo here.

Here also we this is a solution for the charge at anytime t. There are three different cases depending on the value of m.
When b square is greater than k square, b square is less than k square and 3rd case b square equal k square.
For the first case b square greater than k square
it is an overdamped circuit the charge decays exponentially to 0.
The second case, b square less than k square is underdamped circuit.
Q(t) or the charge at any time is given by this relation
e raise to minus bt A e raise to J Omega t + B e raise to minus j omega t.
here m is equal to j omega. Substituting for the values of A, B, e raise to j omega t, e raise to minus j omega t and m and rearranging the terms.

We have for Q(t) equal to Qo e raise in bracket minus R/2L into t k by omega sin omega t plus alfa.
Discharge of the capacitor is damped oscillatory.
The amplitude of oscillatory charge decays as per the term e r – R divided by 2L into t and finally it tends to zero.
Oscillation stops and the capacitor is fully discharged.
The period of damped oscillations T is equal 2 pi by omega.
The current I which is a derivative of dQ by dt is also oscillatory with the same log decrement and
The third case is that b square is equal to k square is a critically damped circuit.
Capacitor charges to the final value zero in the shortest possible time
and the current I(t) is equal to Qo b raise to 2 t e raise to minus bt.

The current stops flowing when the charge reaches its maximum value.
The plot for charge at any time t with respect to t is given for all the three shown for all the three cases here.
For b greater than k, which is also a dead beat case.
Here we have b is equal to critically damped case and oscillatory case.
when b equal to less it is undamped ooscillatory.

The notation used I_L, Ic, I for current
V voltage Q for charge, R for resistor, C capacitor
L is inductor
References my references as shown in the slide
Have three references.
Thank you.