Quadrant II – Transcript and Related Materials

Programme: Bachelor of Science (Second Year)

Subject: Physics

Course Code: PYS 101

Course Title: Network Analysis

Unit: 6 Resonance

Module Name: Series Resonance, Quality factor(Q) and its effect on Bandwidth, Parallel Resonance and Quality factor(Q) of Parallel Resonance

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Notes.

Series resonance, Quality factor (Q) and its effect on Bandwidth, parallel resonance, Q factor of parallel resonance.

Resonance is an important phenomena in electrical circuits. A circuit containing reactance is in resonance if the voltage across the circuit is in phase with the current through it. Series RLC circuit and Parallel RLC circuit produce resonance at a certain frequency of the source voltage and hence are called resonance circuits, and resonance occurring in these circuits are called series resonance and parallel resonance respectively.

Series resonance.

A series RLC circuit is shown in Figure. Assuming all components to be ideal. Let the applied voltage be v(t). If i(t) be the instantaneous current, and applying Kirchoff's voltage law for the closed RLC circuit, we have

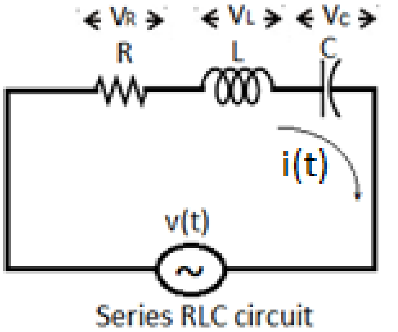
$v(t) = v_R + v_L(t) + v_C(t)$

$v(t) = i(t)R + L\,di(t)/dt + (1/C)\int i(t)dt$

$v_R(t) = i(t)R$

$v_L(t) = L\,di(t)/dt,$

$v_C(t) = (1/C)\int i(t)dt$



Series RLC circuit

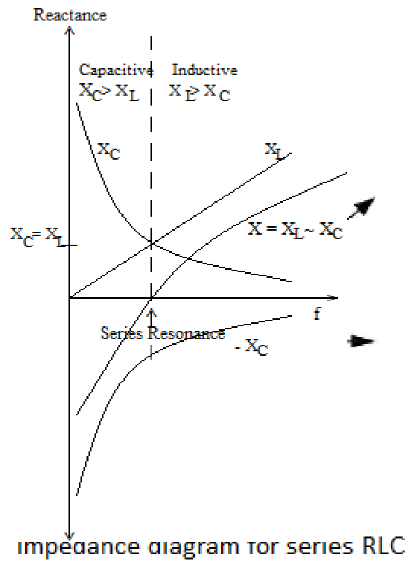
Consider current i(t) to be complex corresponding to complex voltage v(t)
Since the voltage across inductor leads the

current by 90^o and voltage across capacitor lags current by -90^o

$v(t) = i(t)\, Z$ Z – vector impedance

$Z = v(t)/i(t) = R + j(\omega L - 1/\omega C)$

magnitude $|Z| = \sqrt{[R^2 + (\omega L - 1/\omega C)^2]}$



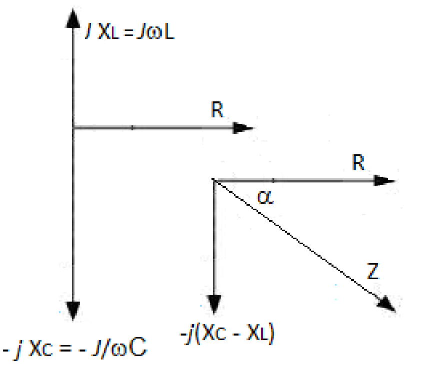
Impedance diagram for series RLC circuit for $X_L > X_C$

The magnitude of circuit current is $i(t) = v(t)/ Z$

If α be the phase angle by which the applied voltage leads/ lags the current or the phase difference between the applied voltage and the current is given by $\alpha = \tan^{-1} (\omega L \sim 1/\omega C)/R$

The impedance $|Z|$, phase angle α, and current $|i(t)|$ vary with frequency of applied voltage. The current i(t) depends on the total impedance Z of the circuit. The current i(t) leads or lags the applied voltage depending upon the values of $X_L = \omega L$ and $X_C = 1/\omega C$. X_L causes current i(t) to lag behind the applied voltage v(t) and X_C causes current i(t) to lead the applied voltage.

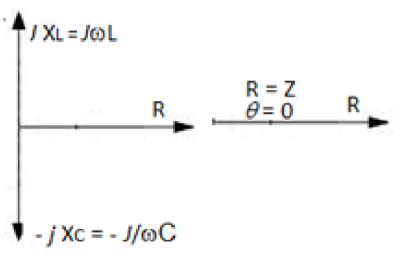
When $X_L > X_C$ the circuit is mainly inductive and when $X_L < X_C$ the circuit is mainly capacitive.



Impedance diagram for series RLC circuit for $X_L < X_C$

When $X_L = X_C$ ($\omega L = 1/\omega C$) the current is in phase with the applied voltage, the net reactance is zero, phase angle $\alpha = 0$, and the circuit is said to be in resonance, with resonance frequency (series resonant frequency).

$\omega_o^2 = 1/LC$ $f_o = 1/2\pi\sqrt{LC}$.



Impedance diagram for series RLC circuit $X_L = X_C$

Z_{min} = R (purely resistive)

⇒ Maximum current, zero phase difference

⇒ Voltages across capacitance and Inductance

are equal in magnitude and opposite in phase

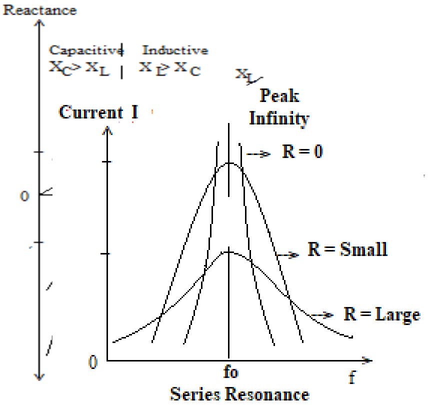
Reactance and Impedance of a series RLC circuit.
Graphical representation

The variations of inductive reactance X_L, capacitive reactance X_C, net reactance $X = X_L \sim X_C$, impedance Z with frequency are shown in Figures.

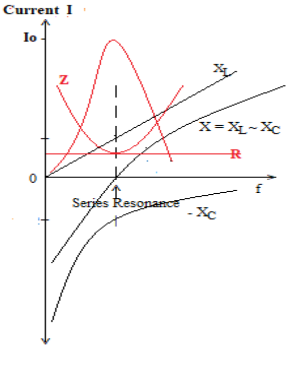
Capacitive reactance X_C (regarded as negative and equal to $1/\omega C$) is inversely proportion to frequency f, Its graph is a rectangular hyperbola (drawn in the fourth quadrant). It is asymptotic to the vertical axis at low frequencies and horizontal axis at high frequencies.
Inductive reactance X_L ($=\omega L = 2\pi fL$) is directly proportional to f, its graph is a straight line passing through the origin.
X_C and Z are infinitely large, capacitor acts as an open circuit and inductive reactance X_L is zero
Resistance R is independent of frequency f and represented by a straight line.
Net reactance $X = X_L \sim X_C$, Its graph is a hyperbola and crosses the X-axis at resonant frequency f_0.

Features of Series Resonance.

1. Resonant frequency $f_0 = 1/2\pi \sqrt{LC}$
(depends on L and C only and independent of circuit resistance R)
2. $Z_{min} = R$ (purely resistive). Phase angle $\alpha = 0$, $v(t) = v_R(t)$,
since $i(t)_{max} = v(t)/Z_{min} = v(t)/R$ R controls the $i(t)_{max}$
3. V_C and V_L are equal in magnitude and 180^o out of phase with each other, the voltage across capacitor and inductor combination is 0 volts.
4. A series resonant circuit is also known as acceptor circuit. This frequency is accepted most readily by the series RLC circuit.



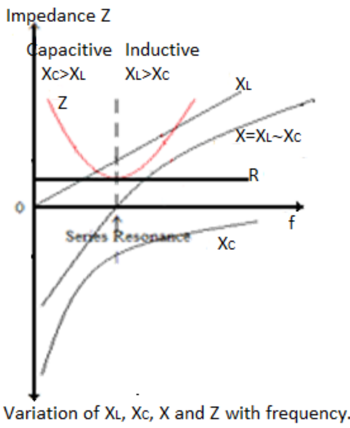
Series Resonance



(At non resonance frequencies combination voltage is always less than the larger individual voltage across either component)
5. Voltage V_C and V_L can be much larger than the source voltage. The ratio V_C/V or V_L/V is called the voltage magnification factor for the series resonant circuit.

Frequency Response of Series RLC circuit.

The frequency response of a series RLC circuit is the graph of the circuit current i(t) or I and the frequency of the source voltage v(t).
The expression for the current at any frequency f is $i(t)=v(t)/Z$.
For a constant v(t), with increase in input frequency from a low to a high value, the current initially rises with the frequency, reaches a peak value and then decreases with the



Variation of XL, Xc, X and Z with frequency.

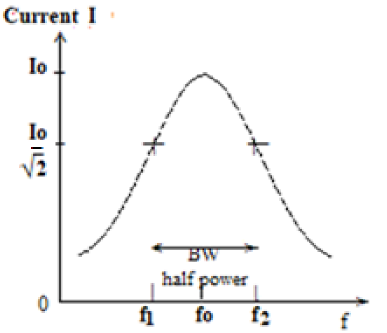
increase in frequency, maximum current $i(t)_{max}$ or say Io occurs at resonance frequency. This curve is known as resonance curve.

Current i(t) varies because impedance Z varies with frequency as reactance X_C and X_L varies with frequency. Due to zero phase difference at resonance entire supply voltage is across R.

The shapes resonance curve for different values of R are shown in Figure. For small values of R, the resonance curve is sharper and such a circuit is said to be sharply resonant or highly selective. For larger values of R, resonance curve is flat and is said to have poor selectivity.
The ability of a resonant circuit to discriminate between one particular frequency and all others is called its selectivity.

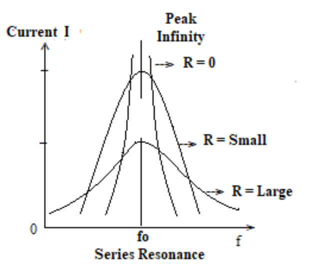
Q-factor of Series Resonant circuit.
Components that store energy (capacitor and inductor) can be described in terms of a quality factor Q. The Q of capacitor or inductors is a ratio of its ability to store energy to the total of all energy losses within the component. i.e. ratio of peak power stored in either the capacitor (P_C) or inductor (P_L) at resonance to the average power that is dissipated by the resistance P_R in the circuit is called the quality factor of the series resonant circuit.
Q-factor $Q = P_C/P_R = P_L/P_R$
$= i_o^2X_{Co}/i_o^2R = i_o^2X_{Lo}/i_o^2R$

$Q = X_{Lo}/R = X_{Co}/R$ $\quad Q = \omega_oL/R = 1/\omega_oCR$



For Ideal R, L and C the Q- factor is equal to voltage amplification factor.

Band width
The variation of current is due to variation in Z with frequency. At resonance the circuit resistance R controls the maximum current. Narrower the bandwidth, higher the selectivity of the circuit and vice versa.



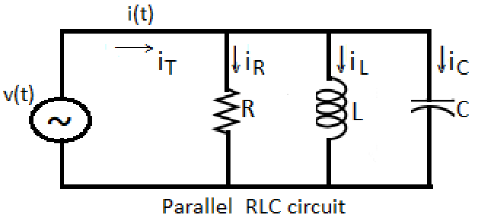
The half power bandwidth of a circuit is given by the band of frequencies which lies between two frequencies f_2 and f_1 (cut-off frequencies) on either side of resonant frequency f_0 where current falls to $i_o/\sqrt{2}$.
The difference between two cut-off frequencies is called the bandwidth (BW) of the series resonant circuit. $BW = \Delta f = f_2 - f_1$

The real power absorbed by RLC circuit is $P = i(t)^2R$,
At resonance the power is $P_0 = i_o^2R$ and at cut off frequencies current is $i_o/\sqrt{2}$.
Power at cut-off frequencies f_1 and f_2 $P_1 = P_2 = i(t)^2R$
$= (i_o/\sqrt{2})^2R = (i_o^2/2)\,R = P_0/2$
Half the power at resonance.

Cut-off frequencies are also called half power frequencies or half power points and the bandwidth is called half-power bandwidth BWhp
$BWhp = \Delta f = f_2 - f_1$
It is also called – 3dB bandwidth.
Impedance Z at cut-off frequencies is $\sqrt{2}R$. ($X_C - X_L = R$)
The f_0 is related to the cut of frequencies f_1 and f_2 by the relation $f_0 = \sqrt{(f_1 f_2)}$
Phase angle α is given by relation $\tan\alpha = (X_L - X_C)/R$.
At resonance phase angle $\alpha = 0$.
At f_1 $X_L - X_C = -R$ $\tan\alpha = -1$ $\alpha = -45^o$, At f_2 $X_L - X_C = R$ $\tan\alpha = +1$ $\alpha = 45^o$

Band width and Q

Quality factor is defined by
$Q = X_{Lo}/R = X_{Co}/R$ $\quad Q = \omega_o L/R = 1/\omega_o CR$
$Q = (1/R)\sqrt{(L/C)}$



Parallel RLC circuit

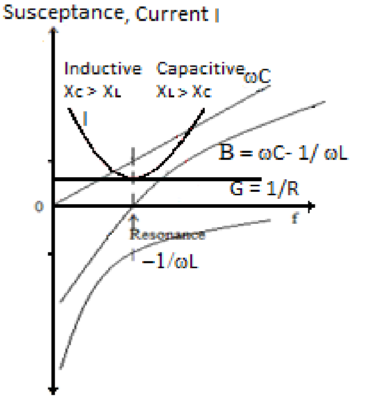
Ratio of resonant frequency by bandwidth gives the quality factor.
$Q = \omega_o / (\omega_1 - \omega_2) = f_o / (f_1 - f_2)$ = Resonant frequency / Bandwidth
1/Q is called selectivity of the given LCR circuit.

Sharpness of Resonance.
Smaller the bandwidth for a given resonant circuit sharper is its response.
Sharpness increases with decrease in bandwidth i.e. with increase in quality factor Q . Selectivity decreases with increase in sharpness.
Sharpness or Q-factor can be increased by increasing R , keeping Land C constant or by keeping R, fo constant and varying L and R

Parallel resonance.

A parallel RLC circuit is shown in Figure. Assuming all components to be ideal.
Let the applied voltage be v(t) and $I_T(t)$ be the instantaneous total current, writing KCL for the parallel RLC circuit.
$i_T = i_R + i_L + i_C$
$i_T(t)$ is the phasor sum of currents. v(t) is the applied voltage.
$i_R = v(t)/R$ is in phase with v(t),

$i_L = v(t)/\,jX_L = -jv(t)/X_L$ lags behind v(t) by 90^o
and $i_C = v(t)/-jX_C = jv(t)/X_C$ leads v(t) by 90^o.



Variation of 1/XL, 1/Xc , B and current I with frequency

$i_T = i_R + i_L + i_C = v(t)/R + -jv(t)/X_L + jv(t)/X_C = v(t)[1/R + j(1/X_C - 1/X_L)$

$i_T = v(t)[1/R + j(\omega C - 1/\omega L)] = v(t)/Z_P$
where Z_P is the total impedance of the parallel RLC circuit.

$1/Z_P = [1/R + j(\omega C - 1/\omega L)] = [1 + jR(\omega C - 1/\omega L)]/R$
$Z_P = R/[1 + jR(\omega C - 1/\omega L)]$
$= [R/(1 + R^2(\omega C - 1/\omega L)^2) - jR^2(\omega C - 1/\omega L)/(1 + R^2(\omega C - 1/\omega L)^2)]$

In terms of real and imaginary part of the impedance $Z_P = R_P - j X_P$
Magnitude of $Z_P = [R_P^2 + X_P^2]^{1/2} = R/\sqrt{}(1 + R^2(\omega C - 1/\omega L)^2$

The magnitude of total circuit current is $i_T = v(t) Z_P = [v(t) \sqrt{}(1 + R^2(\omega C - 1/\omega L)^2]/R$.
α the phase angle by which the applied voltage leads/ lags the current
$\alpha = \tan^{-1} X_P/R_P = \tan^{-1} R(\omega L - 1/\omega C)$

Susceptance and Impedance/ admittance of a parallel RLC circuit.
Graphical representation

$X_C = 1/j\omega C \quad 1/Xc = j\omega C$
$X_L = j\omega L \quad 1/X_L = 1/j\omega L$
$1/X = 1/X_L \sim 1/X_C = j(\omega C - 1/\omega L)$
Three cases arise.
$X_L < X_C$, $\omega L < 1/\omega C$ α negative, i_T is inductive
$X_L > X_C$, $\omega L > 1/\omega C$ α is positive, i_T is capacitive
$X_L = X_C$, $\omega L = 1/\omega C$ --- resonance $f_o = 1/2\pi \sqrt{LC}$
$i_L = i_C$ are equal and opposite and cancel out, $i_T = i_R$
i_T is minimum and in phase with v, purely resistive Zpmax = R
Phase angle $\alpha = \tan^{-1} Xp/Rp$

Frequency Response of parallel RLC circuit
Graph of circuit current i(t) and frequency f of the applied voltage.

$I(t)_T = v/Z_p = v\sqrt{(1+R^2(\omega C-1/\omega L)^2}/R$

$|Zp| = R/\sqrt{(1+R^2(\omega C-1/\omega L)^2}$

v is kept constant

Variation in current is due to change in Z reactance X_L, $X_{C.}$

$X = X_L - X_C = 0$ at resonance $i_{min} = v/R$

Bandwidth (half power bandwidth).

Definition in same way as the case of series circuit, and having halfpower bandwidth. (one can define in terms of circuit impedance)

At bandwidth frequencies, the net susceptance *B* equals the conductance.

Circuit current $i_T = v/Z_T$

$i\ min = v/Zmax$

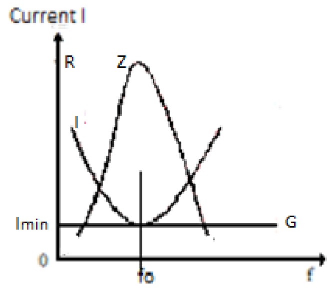
$Z_T = Zmax/\sqrt{2}$

$i = \sqrt{2}\ imin$

$B.W. = \Delta f = f_2 - f_1$

bandwidth of the parallel resonant circuit.

$B.W. = f_o / Q$



Parallel Resonance

Variation of current I with frequency f

Quality Factor

Components that store energy such as capacitors and inductors, can be described in terms of a quality factor Q.

The Q of such a component is a ratio of its ability to store energy to the total of all energy losses within the component.

Q-factor of Series Resonant circuit.

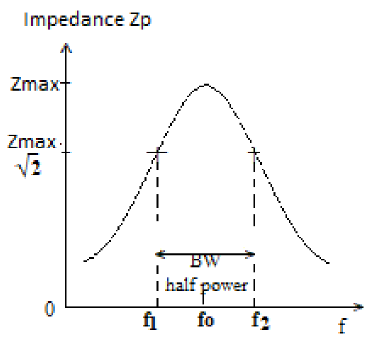
Q = Peak power stored in either the capacitor (P_C) or inductor (P_L) at resonance

/Average (real) power that is dissipated by the resistance P_R

$= P_C/P_R = P_L/P_R$

$= i_o^2 X_{Lo}/ i_o^2 R = i_o^2 X_{Co} / i_o^2 R$

$Q = X_{Lo}/R = X_{Co}/R$ $Q = \omega_o L/R = 1/\omega_o CR$



Variation of Impedance with frequency

For ideal components Q- factor is equal to voltage amplification factor

Parallel RLC circuit is said to be in electrical resonance when the reactive (or wattless) component of current becomes zero. (occurs at resonant frequency)

i.e.at resonance $(\omega L-1/\omega C)= 0$

$X_C = X_L$ i.e $\omega L = 1/\omega C$, $\omega^2 = 1/LC$ $\omega = 1/\sqrt{(LC)}$ and $f_P = 1/2\pi\sqrt{LC}$

phase angle a = 0, and $i_L = i_C$

i_L and i_C are equal and opposite and cancel out. And $i_T = i_R$

i_T is minimum and in phase with applied Voltage V, the circuit is purely resistive and its impedance is maximum $Z_{Pmax} = R$.

Parallel RLC circuit is said to be in electrical resonance when the reactive component of current becomes zero. The frequency at which this happens is known as resonant frequency.

i.e.at resonance $(\omega L - 1/\omega C) = 0$

$X_C = X_L$ i.e. $\omega L = 1/\omega C$, $\omega^2 = 1/LC$ $\omega = 1/\sqrt{(LC)}$ and $f_P = 1/2\pi\sqrt{LC}$

phase angle $\alpha = 0$, and $i_L = i_C$

i_L and i_C are equal and opposite and cancel out. And $i_T = i_R$

i_T is minimum and in phase with v(t), the circuit is purely resistive and its impedance is maximum $Z_{Pmax} = R$.

For an ideal parallel RLC circuit, the parallel resonance frequency is equal to series resonant frequency.

The resonant circuit is purely resistive in both cases and circuit current is in phase with applied voltage.

For an ideal parallel RLC circuit, the parallel resonance frequency is equal to series resonant frequency. The resonant circuit is purely resistive in both cases and circuit current is in phase with applied voltage.

Current at resonance is minimum, hence such a circuit is also known as rejector circuit because it rejects (or takes minimum current of) that frequency to which it resonates.

This resonance is often referred to as current resonance also because the current circulating between the two branches is many times greater than the line current taken from the supply.

The phenomenon of parallel resonance is of great practical importance because it forms the basis of tuned circuits in Electronics