Hello students, welcome to Module number 8 of Unit 2 of Section 1 of the paper classical mechanics and thermal Physics.
The name of this model is nature of orbit - part one.
I'm Yatin Desai, assistant professor inPhysics, Chowgule college, Margao.

In this module you will be learning about the equation of an orbit for a particle moving under an inverse square law force field.
And at the end of this module you will be able to derive the polar equation of an orbit for a particle moving under or inverse square law force field.

So let us start. We start with the equation of an orbit.for a particle moving under an inverse square law force field.

We start with the general equation of an orbit, which is given by d square u by d theta square equals minus m by L square u square f of 1 by u minus u.

To study the motion of particle under the Central Force field which is exhibited by most of the stable systems such as the atomic system, gravitational system, etc. It is interesting to know the shape of the orbit that the particle follows while in motion rather than to solve the equation of motion to explain the development of state of the system with time.

Therefore we transform the equation of motion to the orbit equation using the polar coordinates r and theta.
So hereI have written that orbit equation.
So in this orbit equation M is the mass of the particle, L is the angular momentum of the particle. u is some variable which is related to r by u equal to 1 by r & f of 1 by u is the force.

Now we want to apply this equation of orbit to the inverse square law force.
So we define inverse square law force.
So the inverse square law force is defined as f of r equals k by r square.
Now that K is the constant which is given as minus GMm, where G universal gravitational force constant, capital M times smaller m is the product between two masses. So then it will represent the Newton's law of gravitation.
If K equals q1 q2 divided by 4 pi epsilon not, then it represents the Coulomb’s law.
So K is a constant.

Now to solve the equation of motion we substitute the inverse square law force in the equation of orbit.

Now to do that, we write r as equal to 1 by u. Therefore the inverse square law force is given as f of 1 by u equals k u square. The equation of orbit is now written as d square u by d theta square equals minus m by L square u square times k u square minus u.

Here we can cancel this u square and rearrange this equation to write it as the d square u by d theta square plus u plus mk by L square equals 0.
So we'll call this equation #(1).

So to simplify this further, we write.

u plus mk by L square as equal to y.

And differentiating this with respect to theta, we get du by d theta plus zero
because mk by L square is constant with respect to theta.

And the other side will get d y by d theta.

Differentiating this with once again with respect to theta we get d square u by d theta square equals d square y by d theta square.

Therefore, our previous equation of orbit can now be written as d square y by d theta square plus y equals 0, which we call as equation # (2).

This equation # (2) is 2nd order, nonlinear, homogeneous differential equation.

It is second order because we have y which is differentiated twice with respect to Theta.

It is nonlinear because the 1st order differentiation of y with respect to theta is missing here.

It is homogeneous because the right hand side is equal to 0.

and the solution of this equation (2) is y which is given by
y equal to A times cos of theta minus theta not.

Where A and theta not are arbitrary constants.

Ok, so we have y as A times cos of theta minus theta not as the solution.
And y was written for u plus mk by L square.

So writing it back u plus mk by L square, we get it as equals to A times cos of theta minus theta not.

Rewriting or rearranging this equation we get U equals minus mk by L square plus A cos theta minus theta not

Now to simplify it further, we multiply this equation throughout by minus L square by MK.

So if you do that, we will get the equation as u times minus L square by mk equals 1 minus A L square by mk cos of theta minus theta not.

Again, we had substituted u for 1/r, so if we write it again so we will get it as 1 by r times minus L square by mk equals 1 minus A L square by mk cos of theta minus theta not.

Or we can write it as minus L square by mk whole thing divided by r.equals 1 minus A L square by mk cos of theta minus theta not.

Therefore, we write the above equation as;
l by R equals one plus epsilon cos of theta minus theta not.
We'll call this the equation # (3).

Equation (3) is called the polar equation of a conic section.

And to write equation (3), we have written l for minus L square by mk.
It is called a semi latus rectum which determines the size of an orbit.

Similarly, We have written epsilon for A L square by mk which is called eccentricity,
which determines the shape of an orbit.

Theta not in this equation determines the orientation of an orbit in a plane.

And A determines turning points of motion.

So these are the references, thank you.