Hello students, welcome to the module number 9 of Unit 2 of section I of this paper, classical mechanics and thermal physics. The name of this module is nature of orbit Part 2. I'm Yatin Desai, Assistant Professor in Physics, Chowgule College, Margao.

In this module, you will learn about turning points of motion and nature of orbits and at the end of this module you will be able to analyze turning points of motion, derive an expression for eccentricity, and correlate the total energy possessed by the particle to the shape of the orbit to understand the nature of orbits.

So we start with turning points of motion. We will start with the polar equation of conic section that we have derived in our last module so it was given by l by r equals 1 plus epsilon cos of theta minus theta not. We had called that equation as equation #3. Now, since theta not is an arbitrary constant, we can take theta not to 0. So we can say theta not is arbitrarily taken to 0. That means we can rewrite equation #3 as l by r equals 1 plus epsilon cos of Theta.

We also recollect that we have written that l for minus L square by MK. And epsilon we have written for minus A L square by MK. So. If we substitute it back in the polar equation of conic section, we will get our original equation which was given by 1 by r equals minus MK by L square plus A cos theta.

Now we will talk about the turning points. So number one, the first turning point is when theta equal to zero. And we call that turning point say r = r1. Therefore, using the above equation we get one by r1 as minus MK by L square plus A, because cos of 0 is one. We call this equation as equation #4.

Secondly, we write the second turning point is when theta takes the value π and let us call the turning point is at say r equal to r2. Therefore, we writet from the above equation, 1 by r2 equals minus MK by L square and minus A because cos of Pi is minus one. We call this equation #5.

Next, we also know that at turning points, the total energy of the system is only the effective Potential energy. Because the kinetic energy is zero at the turning points.

That means we can write the total energy E as V effective of r. OK, and that is given as the potential energy due to the central force, which is k by r plus the potential energy due to the centrifugal force, which is given as L ^2 / 2 M r ^2.

Therefore, rearranging this equation we get L ^2 / 2 M R ^2 plus k by r minus E equal to 0. Say we can call this is equation #6. Now this equation #6 is a quadratic equation. It is quadratic, we can say in 1 by r and is of the type of quadratic equation given by ax ^2 + bx + c equal to 0. So we know that the solution for this is given as x = minus b plus or minus root of b ^2 minus 4 ac divided by 2a. So by analogy we can now write the solution of equation 6 as given by 1 by r equals minus b and in place of minus b it's k, so it's minus k plus or minus root of b ^2 which is K ^2 minus 4 a in place of a we have L square by 2 m and in place of c we have minus E. So all things under the under root and divide by 2 times L square by 2m.

So, in this we can cancel this 2 here and we divide 4 by 2 to write 2 here. So, we get the solution as one by r equals - k plus or minus root of k square. So minus of minus will be plus 2 L square E by m divided by L^2 by m.

Or we can write this equation as 1 by r equals m by L square into the bracket minus k plus or minus root of k square plus 2L square E by m. Or we can write this equation as 1 by r equals minus mk by L ^2 plus or minus root of (now) we can take this term inside the square root sign and multiply to both the terms. So to do that we will square it and then multiply to both the terms under the square root sign so it will be m square k square divide by L raised to 4 plus when I multiply it to the second term one of the m’s will get

cancelled with the m in the denominator, so it will be 2 m and L square in the numerator will get cancelled with L to the power four in the numerator, so it will be only 2mE in the numerator divided by L^2.
So we write 2 solutions corresponding to positive sign and negative sign as follows:
OK, so the first solution corresponding to positive sign will call it as 1 by r1 which is minus mk by L^2 plus root of m square k square by L to the power 4 plus 2mE by L square. We'll call this equation #7. And the second solution corresponding to minus sign we write it as: 1 by r2 equals minus mk by L square minus root of $m^2 k^2$ divided by L to the power 4 plus 2mE by L^2 and let us call it as a equation #8.
Now let's compare this equation #8 with equation #4 and 5 that we got previously. So comparing equation 7 and 8 respectively with equation #4 and 5, we can identify the the constant A which was appearing in equation 4 and 5 as equal to square root of m square k square by L to power 4 + 2mE by L square because equation #4 was 1 by r1 equals minus mk by L^2 + A and equation #5 was 1 by r 2 equals minus mk by L square minus A. So we have identified our A. So let us check equation #4 and 5. I'll show you here. So, using this equation, #4 and 5, we can identify the equation for A as the root of m square k square by L to the power er 4 + 2 mE by L^2. OK, once we have identified what is our A we can calculate eccentricity epsilon because it was given by minus A L^2 by mk. In the polar equation of conic section, we have substituted that epsilon as minus AL^2 by mk. So since We now have an expression for A, we can write this equation as, substitute that A which was given as $m^2 k^2$ divided by L to the power 4 +2 mE by L square.
To write the next step will multiply this quantity in the bracket to the quantities inside the square root sign. And before multiplying we will square this and then multiply to each of the terms inside the square root sign. So this negative term which is in this bracket will become positive, minus will become plus because we are squaring it, so it will be multiplied inside as L^2 / m^2 k square. So when we do that, when we multiply L^2 / m^2 k square to the first term, it will be 1. And when we multiply to the second term, OK, 2 will be there, now m in the numerator here will get cancelled with m square, one of the m squares OK in the denominator, so it will be 2 L square E / mk^2.
So this is the equation for eccentricity. Now we know that shape of the orbit is entirely determined by eccentricity. And the above equations showed that eccentricity in turn depends upon the total energy. OK, next, we also know that the total energy E can be positive, negative or 0. OK, therefore the total energy possessed by the particle decides the value of the eccentricity, which in turn decides the shape of the orbit.
So we can now discuss the nature of orbits ok, depending upon the values taken by the total energy. So, we write the eccentricity equation as given by 1 plus $2L^2$ E by mk square and using this we discuss the nature of orbits. In the first column we write the value of the energy. In the second column, we write the value of the eccentricity. And in the third column we write the nature of orbit. If energy is greater than zero, that is, it is positive, then we can see from the equation for an ellipse that epsilon is greater than one and for an orbit, for which epsilon is greater than one, we call that orbit as a hyperbola. Secondly, if energy is 0, then we can see from the equation for an eccentricity that epsilon equals one and in this case the shape of the orbit will be a parabola. Next, if E is negative it is less than 0 but it is greater than the effective value of the (potential energy) minimum value of the effective potential energy, then epsilon will be less than one, but it will be greater than 0. And in this case, the shape of an orbit will be an ellipse. And Lastly. If E itself equals the minimum value of the V effective of r, ok, that minimum value of V effective of r is actually given by minus mk^2 by $2L^2$. So if you substitute here the value of E equals minus mk^2 by

2L^2, we get epsilon to be equal to zero, because it will be root of 1 minus 1, which will be 0. So for epsilon to be zero, eccentricity zero, then the shape of an orbit will be a circle.

OK, so this is how we have discussed the nature of orbits depending upon the values of energy, total energy E which decides the eccentricity. So these are the references, thank you.