Hello students, welcome to Module number 7 of Unit 2 of Section 2 of this paper, Mathematical Physics and Electromagnetic Theory I.

The name of this module is the Electrostatic Potential. I am Yatin Desai, assistant professor in Physics, Chowgule College margao.

In this module, you will learn about the electrostatic potential. At the end of this module, you will be able to express electrostatic potential in terms of the electrostatic field and derive an expression for electrostatic potential due to discrete and continuous distribution of charge.

To start with the electrostatic potential, we begin by the discussion of the electrostatic field at a position r. Now, to give the description of the electrostatic field, I consider a volume element, which is V bounded by the surface S.

We calculate the electric field at some point r, which is the field point. We calculate the electric field at this point r because of the charges present elsewhere. At this point r, we place a test charge q and we calculate the electric field at this point where the test charge q is placed due to the discrete distribution of charges such as q1, q2, q3, q4 as well as the volume distribution of charge because of the volume element here, and the electric field due to the surface element of charge due to this surface element.

Now, when we see the charge plus q and q1, q2, q3, q4 which are placed here, all of these will exert a force on q and the force is given by Coulomb's law. And we can also give the expression for the electric field here by dividing the force by the magnitude of this charge q. So we get the electric field here due to the presence of the other charges.

So the total electric field here due to the discrete distribution of charges such as q1, q2, q3, q4 up to qN is given by the 1st expression here first term in this equation for E(r). So it is 1 by 4 pi epsilon not summation over i which goes from 1 to N qi divided by r minus ri’ square and r minus ri’ cap.

So r minus ri’ cap is the unit vector which specifies the direction along which that electric field is applied.

Secondly, we also calculate the electric field at this point q because of the volume distribution of charge. So for that within this bigger volume V we consider a small volume element dv’ which encloses the charge. We also specify the volume density of charge which is charge per unit volume as rho, so the charge contained within this volume will be rho times the volume of this volume element, which is rho times dv’.

So the electric field at this point r.because of this charge distribution in this volume element. Is given by 1 by 4 pi epsilon not rho times dv’ divided by r minus r’ square and r minus r’ cap.

Now, r minus r’ is actually the separation vector, which is the separation vector between the field point and the source point.

Now, in order to get the total electric field at this point r, we integrate it over the entire volume. That's why we get the second term in the expression for the electric field as one by 4 pi epsilon not integral over the volume v rho r’ dv’ divided by r minus r’ modulus square times r minus r’ cap.

Next we calculate the electric field at point r because of the surface distribution of charge. So for that we can fit a small patch on the surface of area say da’ which is not shown in a diagram. Now that small patch contains a charge which is specified by the surface density of charge which is charge per unit area and we call that as sigma. So the charge contained within that area element da’ is sigma times da’

So we can calculate the electric field at this point r because of the charge contained in area element or because of the charge contained on the entire surface, we get it by integrating that equation. So we get a third term in this equation for electric field as one by 4 pi epsilon not surface integral sigma r’ da’ divided by vector r minus vector r’ modulus square times vector r minus vector r’ cap. This represents the unit vector.

Now, we know that vector r is given by its magnitude and direction. That means vector r is modulus of vector r times r cap. So the unit vector r cap is given by the vector by its magnitude. So if you express the unit vector in the previous equation by the vector divided by its magnitude, so we get that r cap as written as r minus ri’ divided by r minus ri’ magnitude. So that magnitude in the denominator will combine with the previous squared terms and we get it as, r minus ri’ whole modulus cube.

Therefore the equation for the electric field is given as one by four pi epsilon not summation over i which goes from 1 to N, qi modulus of vector r minus vector ri’ cube times vector r minus ri’. So this represents a separation vector plus the field due to volume distribution of charge which is given as one by 4 pi epsilon not integral over V vector r minus vector r’ divided by vector r minus vector r’ cube rho r’ dv’ plus one by 4 pi epsilon not integral over the surface vector r minus vector r’ divided by vector r minus vector r’ cube sigma r’ da’.

Now, we take the curl of equation (1) on both the sides. That means we operate with the operator del cross on both the sides of equation number (1). So on the left hand side we get it as del cross E of r and on the right hand side we get del cross operator operated on vector r minus vector ri’ divided by vector r minus vector ri’ cube and so on. Here we have taken some of the constant quantities such as one by four p epsilon not qi out of that del cross operator.

Now our aim is to solve this equation. To solve this we have to perform the del cross this bracket. This bracket contains the vector and this cube of its modulus, which is a scalar. So to do that, we use one vector identity. The vector identity says that the curl of or del cross phi times A, where phi is the scalar and A is the vector, it is given as phi times curl of A plus gradient of phi cross A

So here we try to use this vector identity to analyze del cross r minus r’ divided by modulus. cube of vector r minus vector r’. Now here, by analogy, we can say that in place of A we have got vector r minus vector r’ and in place of phi, we have modulus cube of vector r minus vector r’. So we get it as equal to 1 divided by vector r minus vector r’ cube del cross vector r minus vector r’ plus del of 1 divided by vector r minus vector r’ cube cross vector r minus vector r’. Now curl of vector r minus vector r’ is 0 because it's a curl of a position vector which is proved to be equal to 0. Just how we have the divergence of position vector proved to be equal to 3, the curl of the position vector is proved to be equal to zero. Therefore, the first term in this equation goes to zero and let us now look more carefully at only the second term. So here we have a del of 1 divided by modulus cube of vector r minus vector r’ and which is crossed with vector r minus vector r’. Let us only analyze this gradient of 1 divided by modulus cube of vector r minus vector r’. So it's like a differential operator which operates on this, so it is minus three times modulus raise to -4 of vector r minus vector r’ times it's a gradient of this scalar which is a vector, so it is represented by the unit vector vector r minus vector r’. Again, it is replaced in terms of the vector divided by its magnitude. So eventually, in total, we get it as -3 vector r minus vector r’ here and this is combined in a denominator to write it as modulus raise to five of vector r minus vector r’. So in this equation, we get this term as -3 times vector r minus vector r’ divided by modulus raise to five of vector r minus vector r’.

Therefore, using that equation (4) in (3), we get it as a cross product between the vector r minus vector r’, and vector r minus vector r’, which is the same vector, so the cross product of both this is zero. Therefore using equation (5) back in equation (2), we prove that curl of the electric field is also equal to 0.

Now, we have one more fact that, curl of gradient of some scalar function is zero, that is del cross del phi is equal to 0. That means there exists a scalar function whose gradient is the electric field. Because, we have curl of E is zero, and here it is curl of the gradient of some scalar function equal to 0. So, that's why equation (6) gives us that E of r is minus gradient of the scalar function U of r where U is called as the electrostatic potential function.

So we can express E of r as the negative gradient of the scalar potential. Now from equation (7), at some point r’, we write it as E of r’ equal to minus gradient of U r’ and taking dot product with dr’ on both sides and then integrating it from some initial reference point to the final point r on both the sides. Reference point here indicates the point where U vanishes. So it is a long enough point where we can say, the reference point may be at Infinity where U is always equal to 0. Therefore we write this equation as integral from initial reference point to the final point, r of E r’ dr’ as equal to this and we can express the right hand side simply as du. And integral of du from the initial reference point to the final reference point. So it's the integral of a differential of a function, is the function itself. So it is U of r with the limits of integration from r reference to r. If r, reference is infinity, then we get it as only U of r. So U of r is actually given as integral of E of r’ dr’. And if we replace that E of r’ as the known expression for E that is given by q divided by 4 pi epsilon not r’ square. If you perform this integral, we get U of r as q by 4 pi epsilon not r. So this expression represents the electrostatic potential due to a point charge.

So with this as a clue, from the equation (1), we guess the potential which gives the electric field as represented by equation (1) is given as U of r as 1 divided by 4 pi epsilon not, summation over i, which goes from 1 to N qi divided by modulus of vector r minus vector ri’ plus 1 / 4 pi epsilon not, volume integral over the volume rho of r’ divided by modulus of vector r minus vector r’ dv’ plus 1 / 4 pi epsilon not surface integral of sigma r’ divided by modulus of r minus r’ da’.

So we get this by analogy from the last equation, so in E of r we had these squared terms here and in U of r we have these r terms with raise to plus one.

So starting from E of r equal to minus del of U of r, it emphasizes the importance of electrostatic potential in determining the electrostatic field.

These are the references for this module. Thank you.