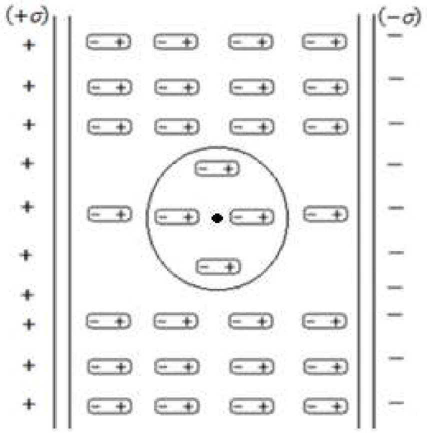
Quadrant II – Transcript and Related Materials

Programme : **Bachelor of Science (Third Year)**
Subject : **Physics**
Paper Code : **PYC 107**
Paper Title : **Mathematical Physics & Electromagnetic Theory - I**
Unit 4 : **Microscopic Theory of Dielectrics**
Module Name : **Molecular field in a dielectric, induced dipoles, A simple model**
Module No : **16**
Name of the Presenter : **Mr. Yatin P. Desai**

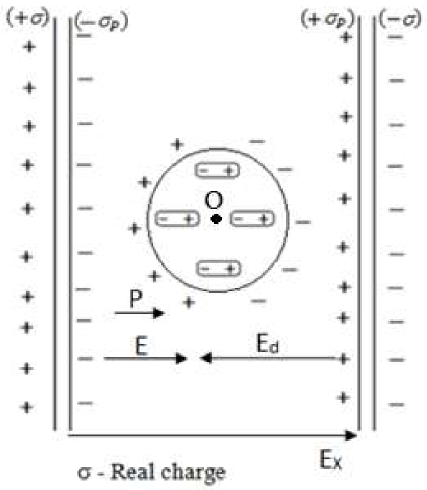
Notes

Molecular Field in a Dielectric:

The electric field that is responsible for polarizing a molecule of the dielectric is called molecular field, $\overrightarrow{E_m}$. It represents field due to all external sources and all polarised molecules of the dielectric except the molecule at the point where the field is to be computed. The molecular field is calculated as follows:



Polarized dielectric



σ - Real charge

σp – Polarization (bound) charge

Consider a thin slab of dielectric placed between two plane parallel metal plates which are oppositely charged. Let a small spherical cavity be cut in the dielectric. The cavity being extremely small as compared to the size of the dielectric but very large compared to molecular size.

Let the cut off partition be put back into the cavity molecule by molecule except the molecule at the centre of the cavity at O, where we want to compute the molecular field.

Let the dielectric be homogeneous, isotropic and linear.

Molecular field at the centre of the cavity is,

$$\overrightarrow{E_m} = \overrightarrow{E_x} + \overrightarrow{E_d} + \overrightarrow{E_s} + \overrightarrow{E'} - - - - - - - - - - - - - - (1)$$

$\overrightarrow{E_x}$: Electric field due to real charges on metal plates.

$\overrightarrow{E_d}$: Electric field due to polarization bound charges on parallel faces of dielectric.

$\overrightarrow{E_s}$: Electric field due to bound charges on spherical surface of cavity.

$\overrightarrow{E'}$: Electric field due to individual dipoles in the cavity.

$$\overrightarrow{E_x} = \frac{\sigma}{\varepsilon_0}$$

$$\overrightarrow{E_d} = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}$$

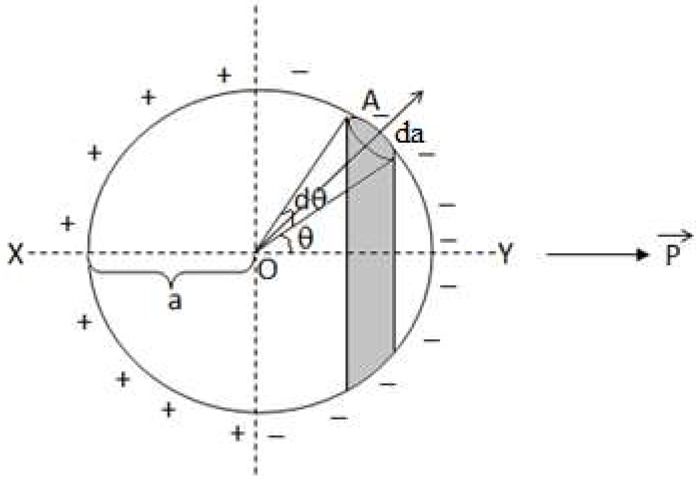
$$\therefore \overrightarrow{E} = \overrightarrow{E_x} + \overrightarrow{E_d}$$

$$i.e. \left|\overrightarrow{E}\right| = \frac{\sigma}{\varepsilon_0} - \frac{P}{\varepsilon_0}$$

Equation (1) becomes;

$$\overrightarrow{E_m} = \overrightarrow{E} + \overrightarrow{E_s} + \overrightarrow{E'} - - - - - - - - - - - - - - (2)$$

To calculate $\overrightarrow{E_s}$:



At X and Y, $\sigma_S = P$

At A, $\sigma_S = P\cos\theta$

Consider a small area (da) at A.

Charge on da = $\sigma_S da$

Electric field at O due to charge $\sigma_S da = \frac{\sigma_S da}{4\pi\varepsilon_0 a^2}$; along OA.

Its rectangular components are,

$\left(\frac{\sigma_S da}{4\pi\varepsilon_0 a^2}\right) cos\theta$; along OY

$\left(\frac{\sigma_S da}{4\pi\varepsilon_0 a^2}\right) sin\theta$; perpendicular to OY

The perpendicular components when summed up for entire circular strip gets cancelled and the horizontal components along OY are added up.

Resultant field at O due to circular strip $= \sum \frac{\sigma_S da}{4\pi\varepsilon_0 a^2} cos\theta$

$$= \frac{\sigma_S cos\theta}{4\pi\varepsilon_0 a^2} \sum da$$

$$= \left[\frac{\sigma_S cos\theta}{4\pi\varepsilon_0 a^2}\right] 2\pi(asin\theta)ad\theta$$

$$= \frac{\overrightarrow{P}\, cos\theta}{4\pi\varepsilon_0 a^2} cos\theta 2\pi a^2 sin\theta d\theta \quad \text{[since } \sigma_S = \overrightarrow{P}\, cos\theta\text{]}$$

$$= \frac{\overrightarrow{P}}{2\varepsilon_0} cos^2\theta sin\theta d\theta$$

Total electric field at O due to entire spherical surface is;

$$\overrightarrow{E_S} = \frac{\vec{P}}{2\varepsilon_0}\int_0^{\pi} cos^2\theta sin\theta d\theta$$ [let cosθ=x, -sinθdθ=dx]

$$\overrightarrow{E_S} = \frac{\vec{P}}{2\varepsilon_0}\int_1^{-1} x^2(-dx) = \frac{\vec{P}}{2\varepsilon_0}\int_{-1}^{1} x^2 dx = \frac{\vec{P}}{2\varepsilon_0}\left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{\vec{P}}{3\varepsilon_0} ------ (3)$$

If all the molecular dipoles in the cavity are assumed to be distributed randomly and all parallel to one another, then,

$$\overrightarrow{E'} = 0 ---------- (4)$$

Substituting equations (3) and (4) in equation (2);

$$\overrightarrow{E_m} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0} ---------- (5)$$

Let $\overrightarrow{P_m}$ be the dipole moment of the molecule.

$\overrightarrow{P_m}$ is proportional to $\overrightarrow{E_m}$

$\therefore \overrightarrow{P_m} = \propto \overrightarrow{E_m}$

$\propto$: **molecular polarisability**

Let N be the number of molecules per unit volume

$$\vec{P} = N\overrightarrow{P_m} = N \propto \overrightarrow{E_m}$$

$$= N \propto \left(\vec{E} + \frac{\vec{P}}{3\varepsilon_0}\right) -------------- (6)$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

We also have, $\vec{D} = \varepsilon\vec{E}$

$$\therefore \varepsilon\vec{E} = \varepsilon_0\vec{E} + \vec{P}$$

$$\therefore \vec{P} = (\varepsilon - \varepsilon_0)\vec{E} = (K\varepsilon_0 - \varepsilon_0)\vec{E} = \varepsilon_0(K-1)\vec{E} ------------- (7)$$

Substituting equation (7) in equation (6);

$$\varepsilon_0(K-1)\vec{E} = N \propto \left(\vec{E} + \frac{\varepsilon_0(K-1)\vec{E}}{3\varepsilon_0}\right)$$

$$\therefore \varepsilon_0(K-1) = N \propto \left[1 + \frac{(K-1)}{3}\right]$$

$$\therefore \varepsilon_0(K-1) = N \propto \left[\frac{K+2}{3}\right]$$

$$\therefore \propto = \frac{3\varepsilon_0}{N}\frac{(K-1)}{K+2} - - - - - - - - - CLAUSIUS\ MOSSOTTI\ RELATION$$

Polar and Non-polar Molecules:

A dielectric may have two types of molecules:

1. **Non-polar molecules:** these are molecules which are polarized only in presence of external electric field.

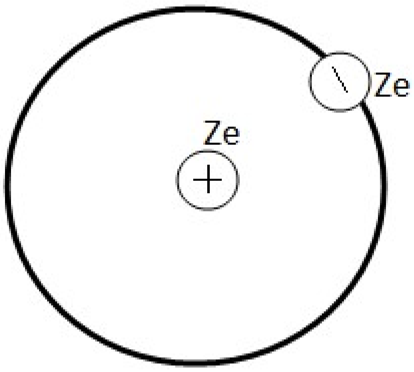
2. **Polar molecules:** these are molecules which have permanent dipole moment even in the absence of a polarising field.

Induced dipoles:

In non-polar molecules, “centers of gravity” of positive and negative charge distribution normally coincide. Symmetrical molecules such as, H_2, N_2, and O_2 or monoatomic molecules such as He, Ne and Ar fall into this category.

The application of electric field causes a relative displacement of the positive and negative charges in non-polar molecules and the ***molecular dipoles created are called Induced Dipoles.***

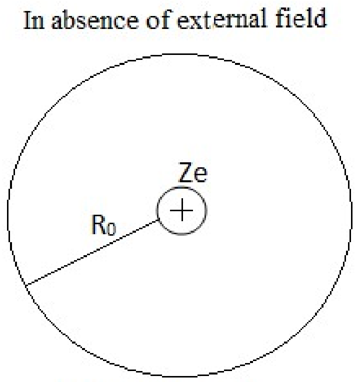
We consider a single neutral atom and use the simple classical model to derive an expression for the induced dipole moment and hence for polarizability. Although this model is designed for monoatomic molecules, it can be used for symmetric diatomic molecules by applying to each of the atoms in the molecule to obtain atomic polarizabilities. The molecular polarizability is then the sum of these, or twice the atomic polarizability.



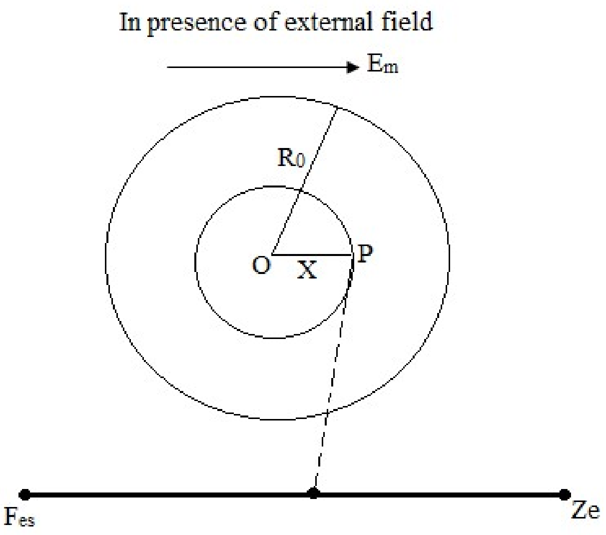
Atom consists of positively charged nucleus with electrons revolving round it. The frequency of revolution of electrons is of the order of 10^{16}or 10^{17} cps. Therefore, negative charge can be considered as spread over the orbit in the form of the ring of negative charge. According to quantum mechanics electrons are not really localized on the orbit but there is a finite probability of electron being located anywhere within the atom. Therefore, a simple classical model of atom would be positively charged nucleus with charge +Ze at the centre of the spherical cloud of negative charge –Ze.

To compute the polarizability of this atom, let the nucleus be assigned the charge Ze, where e is absolute value of the charge and Z is the atomic number. Since the atom is electrically neutral, the total charge in the electron cloud is -Ze.

If the atom is placed in the polarizing field $\overrightarrow{E_m}$, the nucleus will be displaced relative to the center of the charge cloud by distance say 'X'. This displacement will be in the direction of $\overrightarrow{E_m}$. We shall assume that the charge cloud moves rigidly during this displacement i.e. there is no distortion of the cloud by the polarizing field. The displacement x may be determined from the equilibrium of forces on the nucleus; the force $Ze\overrightarrow{E_m}$ acts in the direction of the field, whereas an electrostatic force between the nucleus and the charge cloud tends to restore the initial configuration. By Gauss's law, the negative charge attracting the nucleus is that part of the cloud within the sphere of radius x and if the electron density in the cloud is uniform, then this charge is $\frac{Zex^3}{R_0^3}$. Hence,



R₀ - Atomic radius



F_{es} – Electrostatic force due to negative charge

When an external electric field $\overrightarrow{E_m}$ is applied, nucleus and center of sphere gets separated by a distance say X.

Electric field at P due to shell (radii X and R_0) is zero since P is inside.

Electric field at P due to inner space,

$$\overrightarrow{E'} = \frac{q'}{4\pi\varepsilon_0 X^2}$$

$$\overrightarrow{E'} = \frac{\frac{Ze}{{}^4/_3\pi {R_0}^3}\,{}^4/_3\pi X^3}{4\pi\varepsilon_0 X^2} = \frac{\frac{ZeX^3}{{R_0}^3}}{4\pi\varepsilon_0 X^2} = \frac{ZeX}{4\pi\varepsilon_0 {R_0}^3} \quad along\ PO$$

Therefore. electrostatic force on the nucleus is;

$$\overrightarrow{F_{es}} = Ze\overrightarrow{E'} = Ze\frac{ZeX}{4\pi\varepsilon_0 {R_0}^3}$$

For equilibrium,

$$\overrightarrow{F_{es}} = Ze\overrightarrow{E_m}$$

$$Ze\frac{ZeX}{4\pi\varepsilon_0 {R_0}^3} = Ze\overrightarrow{E_m}$$

$$\frac{ZeX}{4\pi\varepsilon_0 {R_0}^3} = \overrightarrow{E_m}$$

$$ZeX = \overrightarrow{P_m}$$

$$\therefore \overrightarrow{P_m} = 4\pi\varepsilon_0 {R_0}^3\overrightarrow{E_m}$$

$\therefore \propto \overrightarrow{E_m} = 4\pi\varepsilon_0 {R_0}^3 \overrightarrow{E_m}$

$\therefore \propto = 4\pi\varepsilon_0 {R_0}^3$

$but, \propto = \frac{3\varepsilon_0}{N}\frac{(K-1)}{K+2}$ *Clausius Mossotti Relation*

$\therefore \frac{3\varepsilon_0}{N}\frac{(K-1)}{K+2} = 4\pi\varepsilon_0 {R_0}^3$

$\therefore {R_0}^3 = \frac{3}{4\pi N}\frac{(K-1)}{K+2}$: *expression for atomic radii in terms of measurable quantities.*