Quadrant II – Transcript and Related Materials

Programme : **Bachelor of Science (Third Year)**

Subject : **Physics**

Paper Code : **PYC 107**

Paper Title : **Mathematical Physics & Electromagnetic Theory - I**

Unit 4 : **Microscopic Theory of Dielectrics**

Module Name : **Polar molecules, Langevin- Debye formula**

Module No : **17**

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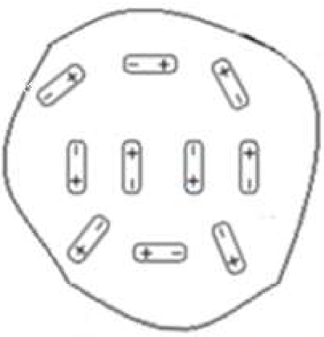
Notes

Polar Molecules in a Dielectric: [The Langevin – Debye Formula]



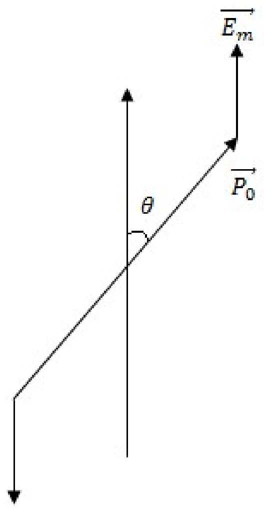
In absence of external electic field

$$\vec{P} = \frac{\sum \overrightarrow{P_m}}{\Delta V}$$

In the absence of external electric field, all dipoles are randomly oriented so that,

$$\sum \overrightarrow{P_m} = 0$$

$$i.e. \vec{P} = 0 \; when \; \overrightarrow{E_{ext}} = 0$$



$\Gamma = P_0 E_m sin\theta$

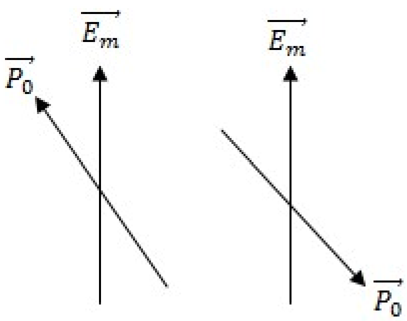
$(P.E.)\omega_P = -P_0 E_m cos\theta$

When external electric field is applied all the molecular dipoles experience torque which try to rotate dipoles and make them parallel to the applied field. At sufficiently strong electric field we expect all the dipoles to be perfectly aligned and polarization $\overrightarrow{P}$ to attain saturation value. But this is not observed in practice with sufficiently strong electric field. Polarization is far from saturation point. This lack of complete alignment is due to thermal energy of molecules which tend to produce randomness in their orientation.

The average effective dipole moment per molecule is calculated using a principle of statistical mechanics which states that at temperature T, the probability of finding a molecule with energy ω is proportional to $e^{-\frac{\omega}{KT}}$, where, $\omega = \omega_K + \omega_P$ [ω_K: K.E. and ω_P: P.E.], K is the Boltzmann constant and T is absolute temperature.

The potential energy of permanent dipole $\overrightarrow{P_0}$ in an electric field $\overrightarrow{E_m}$ is;

$\omega_P = -\overrightarrow{P_0}.\overrightarrow{E_m} = -P_0 E_m cos\theta$; [$\theta$ is an angle between $\overrightarrow{P_0}$ and electric field]

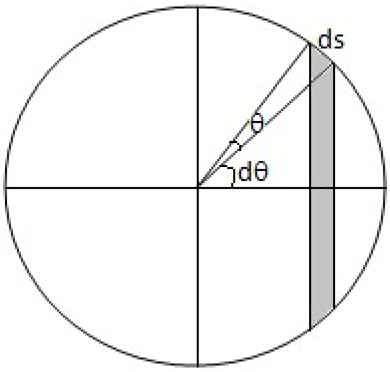


Effective dipole moment of a molecule is its component in the direction of $\overrightarrow{E_m}$ i.e. $P_0 cos\theta$.

It is different for different molecules.

Average value of effective dipole moment;

$$\langle P_0 cos\theta \rangle = \frac{\int P_0 cos\theta e^{-\frac{\omega}{KT}} d\omega}{\int e^{-\frac{\omega}{KT}} d\omega}$$



$$d\omega = elementary\ solid\ angle = \frac{ds}{r^2} = \frac{2\pi(rsin\theta)rd\theta}{r^2} = 2\pi sin\theta d\theta$$

$$\langle P_0 cos\theta \rangle = \frac{\int P_0 cos\theta e^{-\frac{\omega}{KT}} sin\theta d\theta}{\int e^{-\frac{\omega}{KT}} sin\theta d\theta} ------------(1)$$

In equation (1), $e^{-\frac{\omega}{KT}} = e^{-\frac{\omega_k + \omega_p}{KT}}$; $e^{-\frac{\omega_K}{KT}}$ is not a function of θ and therefore it can be taken out of the integral of both numerator and denominator and can be cancelled.

$$\therefore \langle P_0 cos\theta \rangle = \frac{\int P_0 cos\theta e^{-\frac{\omega_P}{KT}} 2\pi sin\theta d\theta}{\int e^{-\frac{\omega_P}{KT}} 2\pi sin\theta d\theta}$$

$\because \omega_P = -P_0 E_m cos\theta$

$$\langle P_0 cos\theta \rangle = \frac{P_0 \int e^{\frac{P_0 E_m cos\theta}{KT}} cos\theta sin\theta d\theta}{\int e^{\frac{P_0 E_m cos\theta}{KT}} sin\theta d\theta} --------(2)$$

$$Put, \frac{P_0 E_m}{KT} = y$$

$$\langle P_0 cos\theta \rangle = \frac{P_0 \int e^{y cos\theta} cos\theta sin\theta d\theta}{\int e^{y cos\theta} sin\theta d\theta} ---------- (3)$$

$Let\ cos\theta = x$

$\therefore -sin\theta d\theta = dx$

When, θ = 0, x=1 and when θ = π, x = -1

Therefore equation (3) can be rewritten as;

$$\langle P_0 cos\theta\rangle = \frac{P_0 \int_{-1}^{1} e^{yx} x(-dx)}{\int_{-1}^{1} e^{yx}(-dx)}$$

$$\therefore \langle P_0 cos\theta\rangle = \frac{P_0 \int_{-1}^{1} e^{yx} x dx}{\int_{-1}^{1} e^{yx} dx} -------------- (4)$$

Integrating by parts:

$$\langle P_0 cos\theta\rangle = P_0 \left[\frac{\left\{\frac{e^{yx}}{y}x\right\}_{-1}^{1} - \int_{-1}^{1} \frac{e^{yx}}{y} 1d}{\int_{-1}^{1} e^{yx} dx}\right]$$

$$\therefore \langle P_0 cos\theta\rangle = P_0 \left[\frac{\left\{\frac{e^{yx}}{y}x\right\}_{-1}^{1}}{\int_{-1}^{1} e^{yx} dx} - \frac{\int_{-1}^{1} \frac{e^{yx}}{y} 1d}{\int_{-1}^{1} e^{yx} dx}\right] = P_0 \left[\frac{\left\{\frac{e^{yx}}{y}x\right\}_{-1}^{1}}{\left\{\frac{e^{yx}}{y}\right\}_{-1}^{1}} - \frac{\int_{-1}^{1} \frac{e^{yx}}{y} 1dx}{\int_{-1}^{1} e^{yx} dx}\right] = P_0 \left[\frac{\left\{\frac{e^{yx}}{y}x\right\}_{-1}^{1}}{\left\{\frac{e^{yx}}{y}\right\}_{-1}^{1}} - \frac{1}{y}\right]$$

$$\therefore \langle P_0 cos\theta\rangle = P_0 \left[\frac{\left\{\frac{e^{y}}{y} + \frac{e^{-y}}{y}\right\}}{\left\{\frac{e^{y}}{y} - \frac{e^{-y}}{y}\right\}} - \frac{1}{y}\right]$$

$$\therefore \langle P_0 cos\theta\rangle = P_0 \left[\frac{\{e^{y} + e^{-y}\}}{\{e^{y} - e^{-y}\}} - \frac{1}{y}\right] --------- (5)$$

$We\ have, e^{i\theta} = cos\theta + isin\theta\ and\ e^{-i\theta} = cos\theta - isin\theta$

$$\therefore cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}\ and\ sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

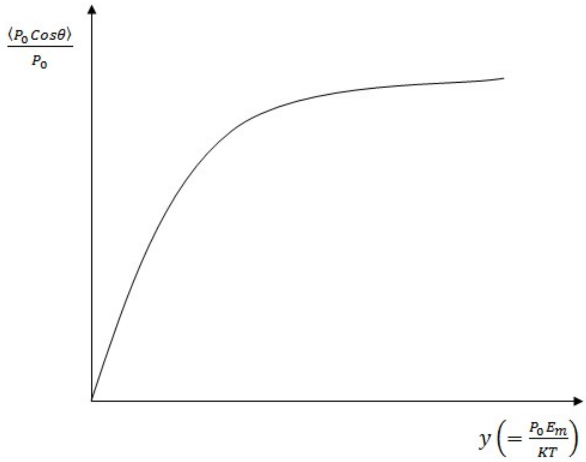
$$cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}\ and\ sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$$

$$coth\theta = \frac{e^{\theta} + e^{-\theta}}{e^{\theta} - e^{-\theta}}$$

Therefore equation (5) is;

$$\langle P_0 cos\theta\rangle = P_0 \left[coth y - \frac{1}{y}\right] -------------- (6)$$

Equation (6) is known as **Lagrangian Formula.**



For most dielectric, P_0 is such that, y is much smaller than 1 [y<<1].

$$coth y = \frac{e^y+e^{-y}}{e^y-e^{-y}} = \frac{\left(1+y+\frac{y^2}{2!}+\frac{y^3}{3!}\right)+\left(1-y+\frac{y^2}{2!}-\frac{y^3}{3!}\right)}{\left(1+y+\frac{y^2}{2!}+\frac{y^3}{3!}\right)-\left(1-y+\frac{y^2}{2!}-\frac{y^3}{3!}\right)}$$

$$\therefore coth y = \frac{2+y^2}{\left(2y+\frac{y^3}{3}\right)} = \frac{2\left(1+\frac{y^2}{2}\right)}{2y\left(1+\frac{y^2}{6}\right)} = \frac{1}{y}\left(1+\frac{y^2}{2}\right)\left(1+\frac{y^2}{6}\right)^{-1}$$

$$\therefore coth y \cong \frac{1}{y}\left(1+\frac{y^2}{2}\right)\left(1-\frac{y^2}{6}\right) \quad [using\ binomial\ theorem]$$

$$\therefore coth y = \frac{1}{y}\left(1-\frac{y^2}{6}+\frac{y^2}{2}-\frac{y^4}{12}\right) = \frac{1}{y}\left(1+\frac{y^2}{3}\right) \cong \frac{1}{y}+\frac{y}{3} ------(7)$$

Substituting equation (7) in equation (6);

$$\langle P_0 cos\theta\rangle = P_0\left[\frac{1}{y}+\frac{y}{3}-\frac{1}{y}\right] = P_0\frac{y}{3}$$

$$\therefore \langle P_0 cos\theta\rangle = {P_0}^2\frac{E_m}{3KT} ----------- (8) \qquad \left[\because y = \frac{P_0 E_m}{KT}\right]$$

Let N be number of molecules per unit volume.

$$\therefore P = N\langle P_0 cos\theta\rangle$$

$$\therefore \frac{P}{N} = \langle P_0 cos\theta\rangle = \frac{{P_0}^2 E_m}{3KT}$$

$$\frac{P}{N} = P_m = \propto E_m$$

$\therefore \propto E_m = \frac{{P_0}^2 E_m}{3KT}$

$\therefore \propto = \frac{{P_0}^2}{3KT} - - - - - - (9) - - - Orientatinal\ polarisability$

Considering non-polar molecules only;

$\propto_0 = 4\pi\varepsilon_0 {R_0}^3 - - - - - - - - - - - (10) - - - - - - Deformation\ polarisability$

$\therefore$ total molecular polarisability of a dielectric is;

$\therefore \propto = \propto_0 + \frac{{P_0}^2}{3KT} - - - - - - - - - - - - - Langevin - Debye\ Equation$