Quadrant II – Transcript and Related Materials

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Unit - 5: Dielectric Properties of Matter

Module Name: Plasma Frequency, Plasmons

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Name of the Presenter: Dr. Manjunath T. Nayak, Assistant Professor of Physics.
Carmel College of Arts, Science & Commerce for Women-Nuvem.

Notes

5.13.1 Plasma Oscillations, Plasma Frequency & Plasmon's

Consider a metal plasma in which a negative charge is introduced at a point chosen as the origin. Let the volume charge density at the point be denoted by ρ. If the charge imbalance because of a momentary fluctuations in charge density lowers the electron concentration at that point by δN, then the resulting charge density at the point is $(\rho - e\delta N)$. The effective electrostatic potential ϕ is related to this charge density by the Poisson equation.

$$\nabla^2\emptyset = \frac{1}{\varepsilon_0}(\rho - e\delta N) \ldots\ldots\ldots\ldots 5.13.1$$

When the excitation energy $e\phi$ is very much smaller than k_BT and the electron concentration N is so low as to permit the use of classical statistics, the decrease in electron concentration can be written as

$$\delta N = N_0\left[exp\left(\frac{e\emptyset}{k_BT}\right) - 1\right]$$

or

$$\delta N \approx N_0\left(\frac{e\emptyset}{k_BT}\right) \ldots\ldots\ldots 5.13.2$$

where N_0 is the equilibrium electron concentration. Putting the value δN from 5.13.2 and 5.13.1, we have

$$\left(\nabla^2 + \frac{N_0e^2}{\varepsilon_0 k_BT}\right)\emptyset = \frac{\rho}{\varepsilon_0} \ldots\ldots\ldots 5.13.3$$

which has the solution,

$$\emptyset = \frac{\rho}{r} exp\left(-\frac{r}{\lambda_D}\right) \ldots \ldots \ldots \ldots 5.13.4$$

with

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T}{N_0 e^2}\right)^{1/2} \ldots \ldots \ldots \ldots 5.13.5$$

Equation 5.13.4 explains the screening of a given electron from other electron *via* the fluctuation of the electron charge density. λ_D is known as the Debye length. The effect of screening is such that when other electrons are at a distance λ_D from the given electron, the normal Coulomb potential is suppressed by the factor (*1/e*). The screening effect closely controls the behaviour of the dielectric constant which is exploited to explain many interesting phenomena involving electron-electron, electron-photon and electron-phonon interactions.

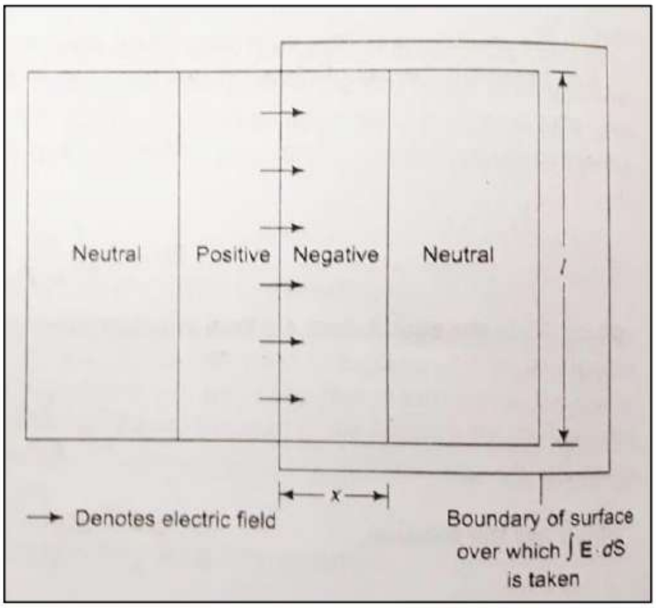
We follow a classical approach to calculate the dielectric constant of a plasma in the absence of an external source of perturbation. Let us consider the two dimensional model of a metal plasma. It is assumed that the configuration of the plasma particles does not depend on their position on the third spatial axis. In view of this fact, we supress this direction in our calculations. On account of a random fluctuation in the equilibrium positions of electrons, two adjoining regions of positive and negative charge densities are created, meaning thereby that the charge neutrality condition is destroyed in these regions. In order to calculate the frequency of a given plasma electron, we work with a picture that is alternative to the one used earlier for describing the plasma oscillations.

Let a small volume element enclosed by surface S is the negatively charged region have a charge q, the electric field in this small volume is given by Gauss theorem:

$$\int_S E.dS = \frac{q}{\varepsilon_0} \ldots \ldots \ldots \ldots .5.13.6$$

And suppressing the 3rd spatial axis, we have

$$\int_S E.dS = -lE \ldots \ldots \ldots \ldots .5.13.7 \qquad (see\ figure\ below)$$



Also,

$$q = -l.x.N_0.e \dots\dots\dots\dots.5.13.8$$

where *x* denotes the displacement or the overshoot in the direction of *E*. from the preceding three relations, we have

$$E = \frac{N_0 e.x}{\varepsilon_0} \dots\dots\dots\dots..5.13.9$$

The electric field *E* serves as a perturbation and drives an electron into oscillatory motion. The equation of motion has the form

$$m\frac{d^2x}{dt^2} = -eE \dots\dots\dots\dots.5.13.10$$

Making use of 5.13.9, we get

$$\frac{d^2x}{dt^2} = -\left(\frac{N_0 e^2}{\varepsilon_0 m}\right)x \dots\dots\dots\dots..5.13.11$$

This relation describes a simple harmonic motion of the characteristic frequency

$$\omega_p = \left(\frac{N_0 e^2}{\varepsilon_0 m}\right)^{1/2} \dots\dots\dots\dots..5.13.12$$

where ω_p is known as the ***plasma frequency***.

In the presence of an external field, both *E* and *x* are bound to a common oscillatory character represented by the time-dependent perturbation ~*exp* (-*iωt*), where ω denotes the angular frequency of the perturbation force.

We may now easily calculate the dielectric constant of the plasma in which the positive ion cores are at rest. Under the influence of *E*, an electron has a dipole moment and the bulk polarization of the plasma is

$$P(\omega) = -N_0 ex \dots\dots\dots.5.13.13$$

With

$$m\frac{d^2x}{dt^2} = -m\omega^2 x = -eE(\omega) \dots\dots\dots\dots 5.13.14$$

But,

$$\varepsilon_r(\omega) = \frac{D(\omega)}{\varepsilon_0 E(\omega)} = 1 + \frac{P(\omega)}{\varepsilon_0 E(\omega)} \dots\dots\dots\dots 5.13.15$$

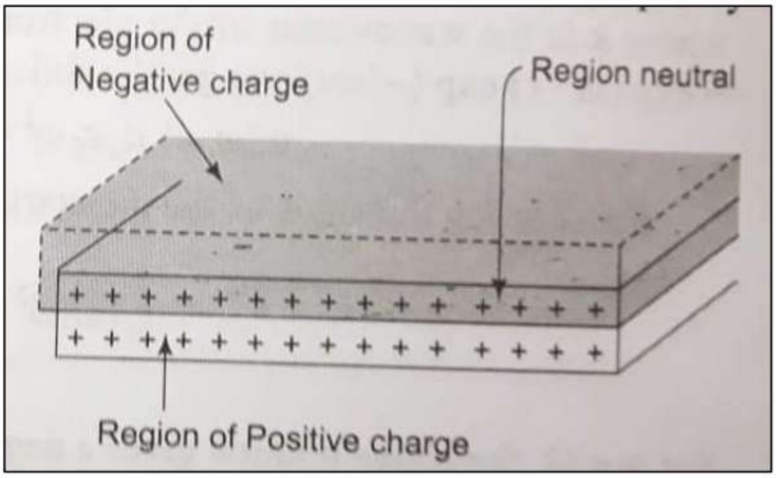
Using 5.13.13 and 5.13.14, we have

$$\varepsilon_r(\omega) = 1 - \frac{N_0 e^2}{\varepsilon_0 m\omega^2} \dots\dots\dots\dots 5.13.16$$

or

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \dots\dots\dots 5.13.17$$

Equation 5.13.17 obviously expresses the dielectric constant of the free electron gas. When the frequency of the position fluctuation ω matches the plasma frequency ω_p, the dielectric constant $\varepsilon_r(\omega) = 0$. This condition refers to the longitudinal plasma oscillations whose wave vector is taken as nearly zero. The energy quanta $\hbar\omega_p$ are commonly known as ***Plasmons***. By the geometry of the longitudinal polarization wave, a depolarization field $E = -P/\varepsilon_0$ is created. This leads to $D = \varepsilon_0 E + P = 0$ and $\varepsilon_r(\omega) = 0$. During a longitudinal plasma oscillation, the electron gas is moved as whole relative to the positive ion background. Figure below shows how the regions of negative charge and positive charge emerge out of the neutral charge distribution within a thin metal slab.



Typical values of Plasmon energy lie between 3 and 20eV. They can be calculated if the electron number density and an appropriate value for the effective mass of electrons are known. The calculated values are found to be in good agreement with the observed ones in several metals and dielectrics. For example, they are 5.71 and 5.95 eV respectively in sodium. The Plasmons may be excited by passing electrons through a thin metal film. The reflection of electrons or photons is also used for the purpose. During the process the charge of an electron coupled with electrostatic field fluctuations of plasma oscillations and the reflected or transmitted electron shows a loss of energy that equals an integral multiple of $\hbar\omega_p$.

If we treated ionic motion independently, we would get a much smaller frequency of oscillation for ions due to their heavier mass. The dielectric constant of the positive ion core background may be defined by

5.13.17 with frequency of ion cores coming in for ω_p. It remains practically constant up to the frequency of perturbation ω well above ω_p. On account of its correspondence with high frequencies, it is denoted by ε_∞. Accordingly, we rewrite 5.13.17 as

$$\varepsilon_r(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2} = \varepsilon_\infty \left[1 - \frac{\omega_p^2/\varepsilon_\infty}{\omega^2}\right]$$

or

$$\varepsilon_r(\omega) = \varepsilon_\infty \left(1 - \frac{\Omega_p^2}{\omega^2}\right) \dots\dots\dots\dots 5.13.18$$

with

$$\Omega_p = \frac{\omega_p}{\sqrt{\varepsilon_\infty}} \dots\dots\dots\dots 5.13.19$$

The frequency Ω_p expresses the uniform collective longitudinal oscillation of the electrons gas against a background of the fixed positive ions. It also denotes low cut-off for the propagation of transverse electromagnetic waves in plasma because at this frequency $\varepsilon_r(\omega) = 0$ $(defining\ the\ longitudinal\ waves)$.