Quadrant II – Notes

Programme: Bachelor of Science (Third Year)

Subject: Physics

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Paper Title: Quantum Mechanics

Unit 1: Waves and particles

Module Name: De Broglie's hypothesis and Review of the Bohr's postulate about stationary states in the light of de Broglie's hypothesis.

Module No: 01

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De Broglie Hypothesis

The wave-particle behaviour of radiation applies to matter as well. A material particle has an associated matter wave that governs its motion. According to de Broglie, for radiation and matter alike, the momentum p is related to the wavelength of the associated wave by the equation

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

m is the relativistic mass, $m = \frac{m_o}{\sqrt{1-v^2/c^2}}$.

Example:

Calculate the de Broglie wavelength for a (a) neutron of kinetic energy 100 MeV and (b) 150g bullet moving at 1000 m/s.

Solution:

(a) The mass of neutron, $m_n = 1.6749 \times 10 - 27$ Kg and 100 $MeV = 1.6 \times 10^{11}\, J$

$$\lambda_p = \frac{h}{p} = \frac{h}{\sqrt{2m_nT}} = \frac{6.626 \times 10^{-34}\, J.S}{\sqrt{2 \times 1.6749 \times 10 - 27 \text{ kg} \times 1.6 \times 10^{11}\, J}} = 2.86 \times 10^{-15} m$$

(b) $\lambda_b = \frac{h}{mv} = \frac{6.626\times10^{-34} J.S}{0.15kg \times1000\, m/s} = 4.42 \times 10^{-36}$

Clearly, the wavelength of the wave associated with the neutron has a same order magnitude as an atomic nucleus and the de Broglie wavelength of the bullet cannot be observed with the most sensitive experimental equipment.

Electron Waves in atoms

Assuming electron goes around in a circular orbit about the nucleus in a hydrogen atom, the centripetal force which holds the electron in the orbit equals the electrostatic force.

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_o}\frac{e^2}{r^2}$$

Simplifying we get

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}}$$

The de Broglie wavelength of this electron

$$\lambda = \frac{h}{mv} = \frac{h}{e}\sqrt{\frac{4\pi\epsilon_0 r_1}{m}} = 33 \times 10^{-11} m \quad -----(1)$$

This wavelength is same as circumference of the electron orbit

$$2\pi r_1 = 33 \times 10^{-11} m$$

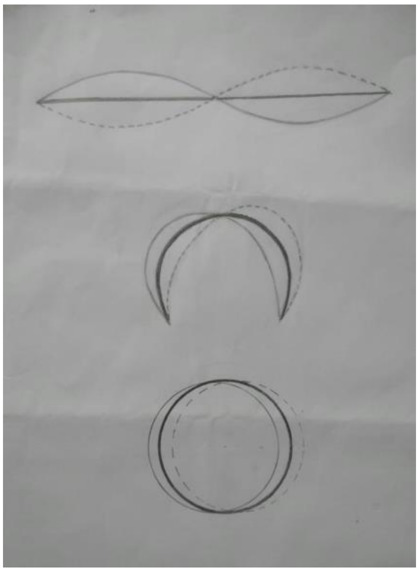

Figure 1: A complete electron de Broglie wave joined on itself.

Assuming the waves associated with electrons in a hydrogen atom is analogous to the vibrations of a wire loop then, an electron can exist in only those orbits in which the circumference of the orbit can contain integral number of de Broglie wavelengths. If a fractional number of wavelengths is placed around the loop then destructive interference will occur and vibrations will die out rapidly.

Hence, the condition for orbit stability is

$$2\pi r_n = n\lambda \qquad ----(2)$$

where r_n is the radius of orbit containing n wavelengths.

We can also obtain the above equation using Bohr quantization of angular momentum

$$L_n = mvr_n = pr_n = nh/2\pi$$

Substituting $p = h/\lambda$ we get the above condition for orbit stability.

Substituting the equation for λ from Eq. (1) in Eq. (2) we get

$$2\pi r_n = \frac{nh}{e}\sqrt{\frac{4\pi\epsilon_0 r_n}{m}}$$

Hence, possible electron orbits are whose radius are given by

$$r_n = \frac{n^2h^2\epsilon_0}{\pi m e^2}.$$

The radius of innermost orbit called the Bohr radius is given as

$$r_1 = a_0 = 5.282 \times 10^{-11} m.$$

The other radii are given by the formula

$$r_n = a_0 n^2.$$

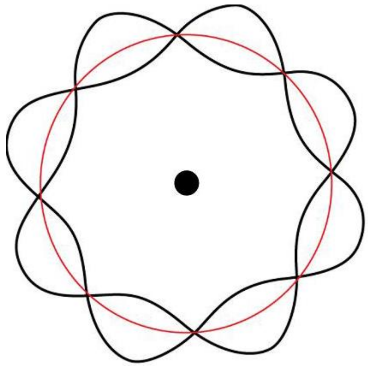

Figure 2: The vibrations of wire loop

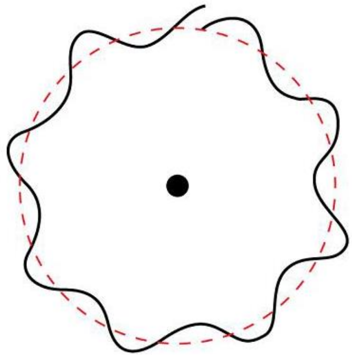

Figure 3: Destructive interference because of fractional number of wavelengths.