Quadrant II – Notes

Programme: Bachelor of Science (Third Year)

Subject: Physics

Paper Code: PYD101

Paper Title: Quantum Mechanics

Unit 1: Waves and particles

Module Name: The concept of particle nature of radiation (Compton Effect)

Module No: 03

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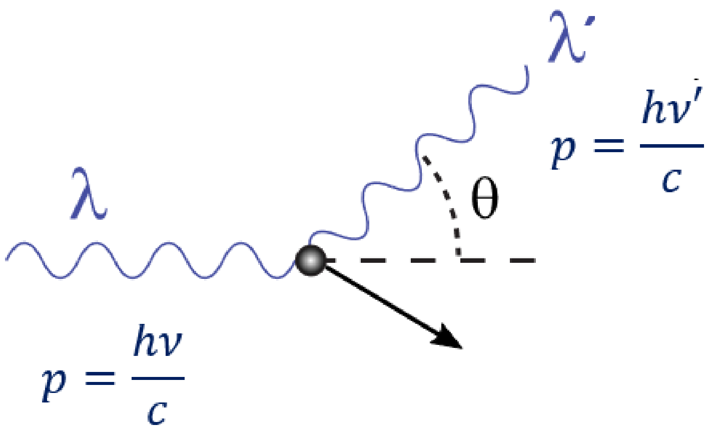
The Compton Effect

Compton in 1925, proposed that the phenomenon of scattering of a photon and an electron is due to an elastic collision between them. According to him, a photon not only carries a quantity of energy $h\nu$, but it also has certain momentum like a material particle. If a photon of energy $h\nu$ strikes an electron it will transfer kinetic energy to the electron and will itself loose energy. The scattered photon will hence have a smaller energy $h\nu'$ and higher wavelength than that of the incident photon. The observed change in frequency or wavelength of a scattered photon is known as the Compton effect.



The Compton Effect

Theory of Compton Effect

If the incident photon has the frequency ν and the scattered photon has the lower frequency ν' then

$$h\nu - h\nu' = T$$

Where, T is the kinetic energy gained by the electron.

From the special theory of relativity, the total energy of the photon is

$$E = pc$$

Where, p is the momentum of the photon.

Therefore,

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

The momentum should be conserved in both perpendicular directions. Therefore, in the x-direction.

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + p\cos\phi$$

And in the y-direction

$$0 = \frac{h\nu'}{c}\sin\theta - p\sin\phi$$

Where ϕ is the angle between initial and recoil electrons and θ is the angle between initial and scattered photon. Multiplying the above two equations by c and rearranging we get,

$$pc\cos\phi = h\nu - h\nu'\cos\theta$$

$$pc\sin\phi = h\nu'\sin\theta$$

Squaring and adding the above two equations we get

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\theta + (h\nu')^2 \qquad \text{----1}$$

From the special theory of relativity the total energy of particles is given by

$E = T + m_0c^2$ and $E = \sqrt{{m_0}^2c^4 + p^2c^2}$

Therefore,

$$(T + m_0c^2)^2 = {m_0}^2c^4 + p^2c^2$$

$$\mathrm{p}^2\mathrm{c}^2 = \mathrm{T}^2 + 2\mathrm{m}_0\mathrm{c}^2\mathrm{T}$$

Hence,

$$p^2c^2 = (hv)^2 - 2(hv)(hv') + (hv')^2 + 2m_0c^2(hv - hv')$$

In the above equation we have used $T = hv - hv'$. Substituting the value of p^2c^2 in Eq. 1 we get

$$m_0c^2(hv - hv') = 2(hv)(hv')(1 - \cos\theta)$$

Dividing the above equation by $2h^2c^2$and using $\nu/\mathrm{c} = \lambda$ we finally get,

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta) \quad - - - 2$$

The above equation gives a change in wavelength of a photon that is scattered through an angle θ by a particle of rest mass m_0. The quantity $\frac{h}{m_0c}$ is Compton wavelength of the scattering particle.