Quadrant II -Transcript

Hello students, welcome to this paper, Quantum mechanics. I am Yatin Desai, assistant professor in Physics, Chowgule college, Margao.
We start with Unit 2, the wave function.
The module number is 7 and the name of the module is representation of a de Broglie wave and velocity of a de Broglie wave.
In this module, you will learn about a representation of a de Broglie wave and velocity of a de Broglie wave.
At the end of this module, you will be able to comprehend the notion of de Broglie wave associated with the particle, the wavefunction, and derive an expression for a de Broglie wave velocity.
We know that electromagnetic radiation such as visible light, x rays, etc behave like a wave and the wave nature is verified using the phenomena of interference, diffraction and polarization.
Electromagnetic radiation also behave like a particle and the particle nature is verified using the experiments such as photoelectric effect, Compton effect.
Thus the electromagnetic radiation has the dual nature; the wave nature and the particle nature.
Louis de Broglie extended the wave particle duality of light to material particles, such as electrons, protons, neutrons, etc.
De Broglie suggested that matter, which is generally treated as particles, also behaves as a wave, and the waves associated with matter are called matter waves or de Broglie waves .
So here we learn about a representation of a de Broglie wave.
The expression for the momentum of a photon of light of frequency nu is given by P equals h nu divided by C, where h is Planck's constant and c is the velocity of light.
Since velocity of Light c is expressed in terms of wavelength and frequency as nu lambda equals c, we can express the momentum in terms of wavelength as p = h by lambda.
The wavelength of a photon is written in terms of momentum in the next equation as lambda equals h by p.
So this equation represents the wavelength associated with the photon. De Broglie asserted that the equation (1) is a completely general formula that applies to photons as well as material particles.
The momentum of a particle of mass m and velocity v is p = m * v and consequently, its de Broglie wavelength from equation (1) could be written as lambda equals h / mv.
So equation (2) is a representation of a de Broglie wave. Thus from equation (2), we see that, greater the particle's momentum, the shorter is its wavelength, because the momentum appears in the denominator of that equation.
In equation (2), we also note that m is the relativistic mass whose expression is given as m equals m not divided by sqrt 1 minus v square by c square.
Equation (2) is verified by experiments involving diffraction of electrons.
So we come from The very nature of particles.
Now in a light wave, the electromagnetic field varies in space and time, and we know that in sound it is a pressure that varies in space and time.

So the question is what is it ,whose variation constitute de Broglie waves?
The variable quantity characterizing the de Broglie waves is called as the wave function.
So the wave function is a quantity whose variations make up the matter waves.
Wave function characterizes the de Broglie wave and it is represented by the Greek letter psi.
The value of the wave function associated with the moving body at a particular point x, y, z in space and at time t is related to the likelihood or the probability of finding the body there at that time.
But that psi in itself has no direct physical significance.
Therefore psi cannot be interpreted in terms of experiment.
We have said that psi represents the probability and about the probability we know that something be somewhere at a given time can have any value between two limits, 0 corresponding to certainty of its absence and 1 corresponding to certainty of its presence.
For instance, probability 0.2 signifies 20% chances of finding the body there at the place at that particular time.
So we have associated a wave with a particle.
It is called as a de Broglie wave.
The wave is represented by the wave function, which is nothing but the probability and has got amplitude which may be positive as well as negative, but the negative probability is meaningless and that is the reason why psi itself cannot be an observable quantity.
But this objection does not apply to another quantity called as the square of the absolute value of the wave function which is termed as modulus square of the wave function.
The probability of experimentally finding the body described by the wave function psi at point x, y, z at time t is proportional to the value modulus of psi square.
It is called the probability density and is given as a product of psi and its complex conjugate psi star.
Because it is a product of psi and psi star, it is always a real and a positive quantity.
Now we call the modulus of psi square as the probability density.
Therefore a large value of modulus of psi square means a strong possibility of the body's presence, while the small value of modulus of psi square means a slight possibility of its presence.
As long as the modulus of psi square is not actually zero, somewhere; however small; there is a definite chance of detecting the body there.
Now let us obtain an expression for the velocity of a de Broglie wave.
Here, since we associate a de Broglie wave with a moving body, we may expect that wave travels at the same velocity v as that of the body.
Let w be the de Broglie wave velocity.
So we write an expression for the wave velocity as w equals nu times lambda. Where nu is the frequency of the wave and lambda is the wavelength of the wave.
Lambda we have already found in an expression, lambda equals h / mv.
Then we can also express the frequency nu in terms of energy E by using the equation E equal to h nu.
That is, nu equals E by h and since E equals m c square, we write the expression for frequency as m c square divided by h.

Now we replace the frequency expression as well as the velocity expression in w as it is given as nu times lambda.
We replace nu as m c square by h and lamba as h by m v.
Consequently we get w as c square by v.
So this is the particle velocity which is obtained as the ratio of c square by v.
c is the velocity of light and v is the velocity of the particle.
Since the particle velocity v cannot be greater than the velocity of light, it follows from the equation #5, that de Broglie velocity w is greater than C!
But this is absurd.
We get an unexpected result which says that the velocity of a wave travels faster than the velocity of light.
We know from this special theory of relativity that no particle can have the velocity equal to the speed of light or greater than the speed of light.
To explain this unexpected result, we have to go into the mathematical representation of a wave and use the notion of wave velocity and group velocity.
So in the next modules, we will see if this velocity expression w is valid.
Does it have any physical significance? Or we have to assign group velocity to the wave group, which is more important.
So we will see this in our next modules.
These are the references. And thank you.