Hello. In this module we will study about the acceptable wavefunction and normalization of the wavefunction. This is model number 12. So first I will discuss about the normalization of the wave function and then little bit about the expectation values. Later on, we'll discuss about the required properties of acceptable wave function. The learner will be able to comprehend what is meant by the normalization of the wave function and explain the meaning of a well behaved wave function in the context of Schrodinger's theory of quantum mechanics. So now we return to the statistical interpretation of the wavefunction. The probability that the particle we found in a range between x and x plus dx is given by P into dx which is equal to mod Psi square x. Now as the particle is going to be somewhere in space, it follows that the integral from minus infinity to plus infinity of Psi square dx should be equal to one. If this is satisfied, if this is true, then the wave function is called the normalized wave function. So even if the wave function is not normalized, if it's square integrable, that is the integral Psi square dx is a finite number, then we can make it normalized by choosing an appropriate constant. That is, if Psi is a solution to the Schrodinger equation, then A Psi is also a solution to the Schrodinger equation where A is some complex constant. Hence we can pick the multiplicative constants so that the above equation is satisfied. So this process is called normalizing the wave function. Now, if the solution to the Schrodinger equation, that is the wave function, is zero or Infinity then the multiplicative constant, no matter what you choose, cannot make the integral equal to one. So these are called non normalizable solutions and cannot represent particles. The state that can be physically realizable corresponds to square integrable solutions to the Schrodinger equation. So the wave function has to be square integrable. If we normalize the wave function at t is equal to 0. It will stay normalised for all future time. This is a very nice property of the Schrodinger equation, so if you normalize it once, that is at t equal to 0, it is ensured that it is normalized at all future times. Without this, the statistical interpretation of quantum mechanics would not hold and the theory would crumble. So the Schrodinger equation automatically preserves the normalization of the wave function that is d by dt of the integral Psi square dx is equal to zero. That implies the integral is constant with respect to time. So now what are the required properties of the wave function other than it being square integrable? Other than it being square integrable, to be an acceptable solution wave function Psi and its derivative dpsi by dx are required to have the following properties, that is Psi and dPsi by dx must be finite. Psi and dPsi by dx must be single valued and Psi and dPsi by dx should be continuous. Now to explain why Psi should be finite, single valued and continuous, I have to briefly explain what is meant by the expectation values. This also will be done in detail in Unit 4 of the course. Here I'll just mention it briefly. So the expectation value is the average of repeated measurements on an ensemble of systems that are prepared Identically and not the average of repeated measurements on one system. So let's say you're measuring the position of the particle and you are measuring position x. So it can take values x1, x2, x3 and so on with probabilities P1, P3, P3 to Pn, then the expectation values defined as x bar which is equal to summation of xi Pi. That is, xi is the possible values of position multiplied by the probability of obtaining this value of position. So we follow this analogy closely in quantum mechanics also. So in quantum mechanics, Pdx is the probability of finding a particle between x and x plus dx, so the average value of position we can write as integral from minus Infinity to plus Infinity of x into pdx. Which is also equal to minus Infinity to plus Infinity of x into Psi* Psi dx which is also equal to minus Infinity to Infinity Psi* x

Psi dx. In this case, the x is sandwiched between Psi* and Psi, so this is the equation which will use and the reason will be apparent when we study unit 4 of this course. In similar way, the expectation value of momentum can be given as p bar is equal to minus infinity to plus infinity Psi* p Psi dx, where p hat is the momentum operator which is given by, p hat equal to minus i h cross partial d by partial dx. Where h cross is h by 2π and h the Planck's constant. So we can write this as minus i h-cross from minus infinity to plus infinity of Psi* partial d Psi by partial dx. In a similar way, we can also write expectation value of total energy. That is equal to integral of Psi* H hat Psi dx. H hat is the total energy operator which is also called the Hamiltonian, which is equal to minus h cross square by two m partial d square by partial dx square plus V. The first term is the kinetic energy operator and next is the potential energy. So we can write the expectation value of total energy in this form over here. Now coming back to the required properties of the wavefunction, Psi must be finite and d Psi by dx also must be finite. Now let's see what happens if psi is not finite. It is infinite. If psi that is the eigenfunction is not finite, then the wave function also wouldn't be finite. That is capital Psi equal to psi (x) into e to the power -iEt by hcross. psi is not finite, so will be the wave function. So if the wave function is not finite then our value of position also won't be finite and same with the value of momentum also won't be finite. So if the wave function is not finite then we may not obtain finite values of position or momentum. This is just an illustration. This is some function f(x) which is plotted with respect to x and it shows at this point function f(x) is not finite. So what will happen if d Psi by dx is not finite?As you can see here the expectation value is not finite, then this will be infinite and then we won't have finite value of momentum in this case. So both psi x and d psi by dx should be finite. Next is psi must be single valued and d Psi by dx also must be single valued. So this is an illustration function f(x) which is plotted with respect to x. As you can see at this point x there are two possible values for f(x) , this one and this one.So let's see what happens if the eigenfunction is not single valued. Of course, if eigenfunction is not single valued then the wave function also won't be single valued. If you go back to this equation of average position or the expectation value of position, we see that for one position you will have two different values of wave function, hence we won't have a definite value of the expectation value of position or the momentum. OK, so similarly. if the d Psi by dx is not single valued, then we'll have two values of d Psi by dx for one value of position, and we won't have a definite expectation value for momentum in that case. So to obtain a definite value of position or momentum psi and d psi by dx must be single valued. Finally, psi x must be continuous and d psi by dx must also be continuous. So now this is a function f(x) plotted with respect to x. So as you can see at this point there is a discontinuity. So what will happen if psi is not continuous? If psi is not continuous, then this implies that d psi by dx will be infinite. So again, if d psi by dx is infinite, then we cannot obtain a finite value of momentum. As you can see here and if d psi by dx is not continuous, that implies the d square psi by d x square will be infinite. In that case we cant get a finite value of total energy because you can see there's a term here d square psi by dx square. Also the Schrodinger equation itself has that term. So this is the time independent Schrodinger equation , so there is this d square psi by dx square term which would be infinite if d psi by dx is not continuous. So to summarize, the wave function should be square integrable, sould be finite, single valued and continuous. Also d psi by dx also should be single valued, finite and continuous. These are some important references. Thank you.